> The following is a general practice test for the algebra placement exam. This pre-test is not necessarily complete or comprehensive of all exam topics. Unlike this practice test, the placement exam is multiple-choice. Students are encouraged to work through these examples before consulting the solutions.

1. Solve for x: $\log_4\left(\frac{8}{x}\right) = 2$

> **Solution:** Re-write the equation: $\left(\frac{8}{x}\right) = 4^2 = 16$. Solve this for x: $x = \frac{1}{2}$.

2. Find $f^{-1}(x)$, the inverse function of $y = f(x) = 3x + 4$.

> **Solution:** Switch y and x and solve for y:
>
> $$\begin{aligned} x &= 3y + 4 \\ \Rightarrow 3y &= x - 4 \\ \Rightarrow y &= f^{-1}(x) = \frac{1}{3}x - \frac{4}{3} \end{aligned}$$

3. Solve the following equation for x. $\sqrt{x+14} - x = 2$.

> **Solution:** Add x to both sides:
>
> $$\sqrt{x+14} = x + 2$$
>
> Square both sides:
>
> $$x + 14 = (x+2)^2 = x^2 + 4x + 4$$
>
> Combine terms:
>
> $$x^2 + 3x - 10 = 0$$
>
> Factor:
>
> $$(x-2)(x+5) = 0$$
>
> So possible solutions are $x = 2$, $x = -5$. If we plug these values into the original equation, we see that $x = 2$ satisfies the equation, while $x = -5$ does not. So only the solution $x = 2$ remains.

4. Write the following complex number in the standard form $a + bi$. $\frac{6-i}{1+i}$

Solution: The conjugate of the denominator is $1 - i$. Multiply by the expression by $1 = \dfrac{1-i}{1-i}$

$$\left(\frac{6-i}{1+i}\right)\left(\frac{1-i}{1-i}\right) = \frac{(6-i)(1-i)}{(1+i)(1-i)} = \frac{6-6i-i+i^2}{1-i+i-i^2} = \frac{6-7i+i^2}{1-i^2}$$

Recall that $i = \sqrt{-1}$, so $i^2 = -1$:

$$= \frac{6-7i-1}{1+1} = \frac{5-7i}{2} = \left(\frac{5}{2}\right) + \left(-\frac{7}{2}\right)i$$

This is standard form, with $a = \dfrac{5}{2}$, and $b = -\dfrac{7}{2}$.

5. Solve the following equation for x, in the complex number system: $10x^2 + 6x + 1 = 0$

Solution: For a quadratic equation of the form $ax^2 + bx + c = 0$, we can write the solutions for x using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ In our equation, $a = 10$, $b = 6$, $c = 1$. Plug these values into the quadratic formula:

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(10)(1)}}{2(10)} = \frac{-6 \pm \sqrt{36-40}}{20} = \frac{-6 \pm \sqrt{-4}}{20}$$

Rewrite $\sqrt{-4}$ using $i = \sqrt{-1}$

$$= \frac{-6}{20} \pm \frac{\sqrt{4}\sqrt{-1}}{20} = \frac{-3}{10} \pm \frac{2i}{20} = -\frac{3}{10} \pm \frac{1}{10}i$$

So our solutions are $x = -\dfrac{3}{10} + \dfrac{1}{10}i$, $x = -\dfrac{3}{10} - \dfrac{1}{10}i$

6. Let $z = 3 - 4i$. Find $z\bar{z}$, where $\bar{z}$ refers to the complex conjugate of z.

Solution: $\bar{z} = 3 + 4i$, so $z\bar{z} = (3-4i)(3+4i) = 9 + 12i - 12i - 16i^2 = 9 - 16i^2$. Recall $i^2 = -1$: $z\bar{z} = 9 + 16 = 25$

7. If $y = f(x) = x^2 - 4x + 2$, find the cartesian coordinates of the vertex of the parabola defined by the graph of $f(x)$.

Solution: If we write the function in the form $y = (x-k)^2 + h$, then the vertex occurs at the point (k, h). To do this, we will complete the square:

$$\begin{aligned} y &= x^2 - 4x + 2 \\ &= x^2 - 4x + (4-4) + 2 \\ &= (x^2 - 4x + 4) - 2 \\ &= (x-2)^2 - 2 \end{aligned}$$

So the vertex occurs at the point $(2, -2)$.

8. Find all real zeros of the following function: $f(x) = x^3 + 2x^2 - 5x - 6$ (all real solutions x to the equation $f(x) = 0$).

Solution: The rational root theorem states that for a polynomial of the form $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0 = 0$, every rational root $x = \pm\frac{p}{q}$ must satisfy the following: p is an integer factor of the constant a_0, and q is an integer factor of a_n. Moreover, we know that such an n^{th}-degree polynomial has at most n real roots.

For the polynomial in question, $a_n = 1$, and $a_0 = -6$. So the candidates for rational roots are: $x = \pm 1, \pm 2, \pm 3, \pm 6$. By substituting into the original equation, we find that $x = -3, -1$, and 2 satisfy the equation. Since the polynomial is of degree 3, we know that it has at most 3 real roots, which are -3, -1, and 2.

9. Solve the following equation for x. $5^{1+3x} = \frac{1}{5}$

Solution: Take the logarithm of both sides:

$$\log_5\left(5^{1+3x}\right) = \log_5\left(\frac{1}{5}\right)$$

Left side:

$$\begin{aligned}\log_5\left(5^{1+3x}\right) &= (1+3x)\log_5(5) \\ &= 1+3x\end{aligned}$$

Right side:

$$\begin{aligned}\log_5\left(\frac{1}{5}\right) &= \log_5(1) - \log_5(5) \\ &= 0 - 1\end{aligned}$$

Simplified equation:

$$\begin{aligned}1+3x &= -1 \\ \Rightarrow x &= -\frac{2}{3}\end{aligned}$$

10. Solve for x in the following equation. $\log_x(3) = \log_6(36)$

Solution: Right side:

$$\begin{aligned}\log_6(36) &= a \\ \Rightarrow 6^a &= 36 \\ \Rightarrow a &= 2\end{aligned}$$

Simplified equation:

$$\log_x(3) = 2$$

$$\begin{aligned} \Rightarrow x^2 &= 3 \\ \Rightarrow x &= \sqrt{3} \end{aligned}$$