The following is a general practice test for the trigonometry placement exam. This pre-test is not necessarily complete or comprehensive of all exam topics. Unlike this practice test, the placement exam is multiple-choice. Students are encouraged to work through these examples before consulting the solutions.

1. Solve the following equation for θ, subject to the constraint $0 \leq \theta < 2\pi$. $\tan^2\theta = \sqrt{3}\tan\theta$

Solution: Factor the equation:

$$\tan\theta\left(\tan\theta - \sqrt{3}\right) = 0$$

Case I: $\tan\theta = 0$, which has solutions $\theta = 0,\ \pi$ on the given interval.

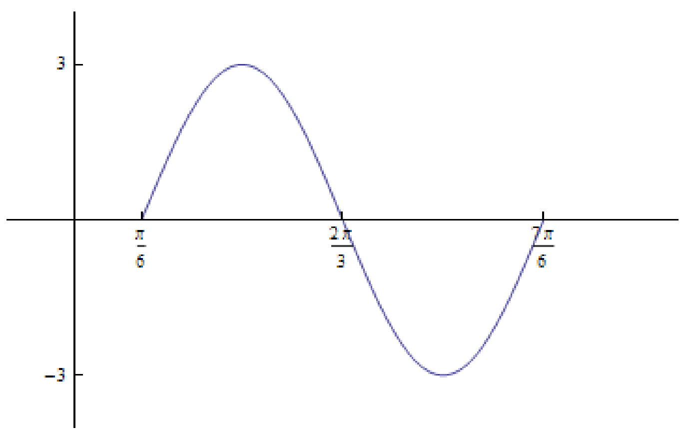
Case II: $\tan\theta = \sqrt{3}$, and $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$ solve this equation on the given interval.

Solutions on the interval are $\theta = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$.

2. Graph the following function over one period: $y = f(x) = 3\sin\left(2x - \frac{\pi}{3}\right)$

Solution: For a function of the form $y = f(x) = A\sin(kx + b) + c$, we can identify the amplitude $|A|$, the period $\frac{2\pi}{k}$, the phase shift $-\frac{b}{k}$, and the vertical shift c.

Our particular function has amplitude 3, period π, phase shift $\frac{\pi}{6}$, and vertical shift 0. Below is a graph of the function.



3. Find the *exact* value of the following expression: $\sin\left(\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5}\right)$

Solution: Let $u = \cos^{-1}\frac{5}{13}$ and $v = \cos^{-1}\frac{4}{5}$. Then $\cos u = \frac{5}{13}$ and $\cos v = \frac{4}{5}$. Now apply the right-triangle definition of the cosine and sine functions, using the Pythagorean theorem:

$$\sin u = \frac{\sqrt{13^2 - 5^2}}{13} = \frac{\sqrt{144}}{13} = \frac{12}{13}$$
$$\sin v = \frac{\sqrt{5^2 - 4^2}}{5} = \frac{\sqrt{9}}{5} = \frac{3}{5}$$

Using this information, we calculate the original expression using the difference rule for the sine function:

$$\begin{aligned} \sin(u - v) &= \sin u \cos v - \cos u \sin v \\ &= \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) \\ &= \frac{48}{65} - \frac{15}{65} \\ &= \frac{33}{65} \end{aligned}$$

4. Calculate the *exact* value of $\phi = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Solution: The restriction on the function $\tan^{-1}(x)$ implies $\tan\phi = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$ on the interval $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$. Using a reference triangle in quadrant IV, the solution is $\phi = -\frac{\pi}{6}$.

5. Reduce the following expression to a single trigonometric function: $\frac{1 - \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 - \cos\theta}$

Solution: Combine fractions with a common denominator, and simplify:

$$\begin{aligned} \frac{1 - \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 - \cos\theta} &= \frac{(1 - \cos\theta)^2 + \sin^2\theta}{\sin\theta\,(1 - \cos\theta)} \\ &= \frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta\,(1 - \cos\theta)} \\ &= \frac{1 - 2\cos\theta + 1}{\sin\theta\,(1 - \cos\theta)} \\ &= \frac{2\,(1 - \cos\theta)}{\sin\theta\,(1 - \cos\theta)} \\ &= \frac{2}{\sin\theta} \\ &= 2\csc\theta \end{aligned}$$

6. Find all values of θ on the interval $0 \le \theta < 2\pi$ that satisfy the following equation: $\cos 2\theta - 3\sin\theta = 2$

Solution: Rewrite using identity $\cos 2\theta = 1 - 2\sin^2\theta$:

$$1 - 2\sin^2\theta - 3\sin\theta = 2$$
$$\Rightarrow 2\sin^2\theta + 3\sin\theta + 1 = 0$$
$$\Rightarrow (2\sin\theta + 1)(\sin\theta + 1) = 0$$

Case I: $\sin\theta = -\frac{1}{2}$, with solutions $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ on the given interval.

Case II: $\sin\theta = -1$, with solution $\theta = \frac{3\pi}{2}$ on the given interval.

Solutions on the interval are $\theta = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$.

7. Find the *exact* value of the following expression: $\cos\left(2\tan^{-1}\frac{4}{3}\right)$

Solution: Let $\alpha = \tan^{-1}\frac{4}{3}$. Then $\tan\alpha = \frac{4}{3}$, and using a right triangle and Pythagoras' theorem, we can say:

$$\sin\alpha = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$$
$$\cos\alpha = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$$

Now re-write the original expression using a double-angle cosine identity:

$$\begin{aligned}\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25}\end{aligned}$$

8. Find all solutions for ϕ on the interval $0 \le \phi < 2\pi$, for the given equation: $\cos\phi = \sec\phi$

Solution: Rewrite and simplify equation:

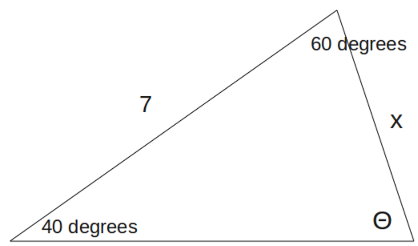
$$\cos\phi = \frac{1}{\cos\phi}$$
$$\Rightarrow \cos^2\phi - 1 = 0$$
$$\Rightarrow (\cos\phi + 1)(\cos - 1) = 0$$

Case I: $\cos\phi = -1$, with solutions $\phi = \pi$ on the given interval.

Case II: $\cos\phi = 1$, with solutions $\phi = 0$ on the given interval.

Solutions on the interval are $\phi = 0, \pi$.

9. Refer to the following figure to calculate the length of side x to two decimal places



> **Solution:** We will use the law of sines: $\dfrac{\sin 40^\circ}{x} = \dfrac{\sin\theta}{7}$. Because the interior angles of any triangle must sum to 180°, $\theta = 180^\circ - 40^\circ - 60^\circ = 80^\circ$ Solving the previous equation for x yields $x = \dfrac{7\sin 40^\circ}{\sin 80^\circ} \approx 4.57$

10. On a given (not necessarily right) triangle, the following is true: $a = 3$, $b = 4$, and $\gamma = 40^\circ$. Find the length of side c accurate to two decimal places.

> **Solution:** According to usual conventions, γ is the angle opposite side c. Hence we will use the law of cosines:
>
> $$\begin{aligned} c = \sqrt{a^2 + b^2 - 2ab\cos\gamma} &= \sqrt{(3)^2 + (4)^2 - 2(3)(4)\cos\gamma} \\ &= \sqrt{25 - 24\cos 40^\circ} \\ &\approx 2.57 \end{aligned}$$