10/13/12

1 Questions:

2 Converge or Diverge:

1. $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+2}\right)$

2. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln(\ln(n))}{\ln(n)}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n!(2n)!6^n}{(3n)!}$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^5+n+1}}{\sqrt[4]{2n^{18}+n^7+4n+2}}$

5. $\displaystyle\sum_{n=3}^{\infty} \frac{\cos(e^{n!})}{n^2+1}$

3 Power Series:

Find the radius of convergence and interval of convergence for the power series

1. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$ We apply the ratio test:

$$\begin{aligned} &\lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!}\frac{n!}{x^n}\right| \\ &= \lim_{n\to\infty} \left|\frac{xx^n n!}{(n+1)n!x^n}\right| \\ &= \lim_{n\to\infty} \left|\frac{x}{n+1}\right| \\ &= |x| \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1. \end{aligned}$$

Therefore the series converges for all values of x. Radius of convergence is $[0, \infty)$ and interval of convergence is all real numbers: $(-\infty, \infty)$. Power series has center 0.

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$ We apply Ratio Test:

$$\lim_{n\to\infty} \left|\frac{(x-2)^{n+1}}{(n+1)^2+1}\frac{n^2+1}{(x-2)^n}\right| = \lim_{n\to\infty} \left|\frac{(x-2)(x-2)^n(n^2+1)}{[(n+1)^2+1](x-2)^n}\right|$$
$$= |x-2| \lim_{n\to\infty} \frac{n^2+1}{(n+1)^2+1} = |x-2| < 1$$

$$-1 < x-2 < 1 \Rightarrow 1 < x < 3$$

So the radius of convergence is 1. Is interval of convergence $(1,3)$? We need to check when $|x-2| = 1$. If $x = 1$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$$

Convergent by test for absolute convergence or by alternating series test. Check $x = 3$:

$$\sum_{n=0}^{\infty} \frac{1}{n^2 + 1}$$

Convergent by limit comparison test (compare to $\sum \frac{1}{n^2}$).

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n3^n}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{n(2x - 1)^{2n}}{(n^2 + 1)5^n}$