1 Questions:

2 Power Series:

Find radius and interval of convergence and center of the power series.

1. $\sum_{n=8}^{\infty} n^7 x^n$ Ratio Test:

$$\lim_{n\to\infty}\left|\frac{(n+1)^7x^{n+1}}{n^7x^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^7xx^n}{n^7x^n}\right|$$
$$= |x|\lim_{n\to\infty}\frac{(n+1)^7}{n^7} = |x|$$

Need $|x| < 1$, $-1 < x < 1$. Center is at $c = 0$ and radius is $R = 1$. Check endpoints. $x = -1:$

$$\sum_{n=8}^{\infty}(-1)^n n^7$$

diverges by the divergence test. $x = 1$:

$$\sum_{n=8}^{\infty} n^7$$

Diverges. Therefore the interval of convergence is $I = (-1, 1)$.

2. $\sum_{n=0}^{\infty}\frac{8^n}{n!}x^n$ Ratio Test:

$$\lim_{n\to\infty}\left|\frac{8^{n+1}x^{n+1}}{(n+1)!}\frac{n!}{8^nx^n}\right| = \lim_{n\to\infty}\left|\frac{8\cdot 8^n\cdot x\cdot x^n\cdot n!}{(n+1)n!8^nx^n}\right|$$
$$= \lim_{n\to\infty}\left|\frac{8x}{n+1}\right|$$
$$= 8|x|\lim_{n\to\infty}\frac{1}{n+1} = 0 < 1.$$

Ratio test implies the power series is absolutey convergent for all values of x. Radius of convergence $R = \infty$ and $I = (-\infty, \infty)$. Center is $c = 0$.

3. $\sum_{n=12}^{\infty} e^n(x-2)^n$ Root Test:

$$\lim_{n\to\infty}|e^n(x-2)^n|^{1/n} = \lim_{n\to\infty}|e(x-2)| \quad = e|x-2| < 1$$

$|x-2| < 1/e$. Radius of convergence is $R = 1/e$. $-1/e < x-2 < 1/e \Rightarrow 2 - 1/e < x < 2 + 1/e$.

$$\sum_{n=12}^{\infty} e^n(2-1/e-2)^n = \sum_{n=12}^{\infty} e^n(-1/e)^n = \sum_{n=12}^{\infty}(-1)^n$$

Test for divergence implies series diverges for $x = 2 - 1/e$.

4. $\sum_{n=1}^{\infty}\frac{x^n}{\ln(n)}$

5. $\sum_{n=1}^{\infty}\frac{(-5)^n(x-3)^n}{n^2}$

6. $\sum_{n=0}^{\infty}\frac{x^n}{n^4+2}$