

1 Questions:

2 Problems:

1. Eliminate the parameter for the parametric curve defined by $x = t^{-1}$ and $y = t^{-2}$.
2. Sketch the curve $c(t) = (t^2, t^3 - 3t)$.
3. Find the equation of the tangent line for the parametric curve defined by $x = 6 \sin(t)$ and $y = t^2 + t$ at the point $(0, 0)$.
4. Find the length of the curve defined by $x(t) = e^t + e^{-t}$ and $y(t) = 5 - 2t$ on the interval $0 \leq t \leq 3$.

$$\begin{aligned} L &= \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt \\ \frac{dx}{dt} &= e^t - e^{-t} \quad \frac{dy}{dt} = -2 \\ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (e^t - e^{-t})^2 + 4 = e^{2t} - 2e^t e^{-t} + e^{-2t} + 4 \\ &= e^{2t} - 2 + e^{-2t} + 4 \\ &= e^{2t} + 2 + e^{-2t} \\ &= (e^t + e^{-t})^2 \\ L &= \int_0^3 \sqrt{(e^t + e^{-t})^2} dt \\ &= \int_0^3 (e^t + e^{-t}) dt \\ &= [e^t - e^{-t}]_0^3 \\ &= e^3 - e^{-3} - (1 - 1) = e^3 - e^{-3} \end{aligned}$$

5. Find the length of the curve defined by $x(t) = \frac{t}{1+t}$ and $y(t) = \ln(1+t)$ for $0 \leq t \leq 2$.