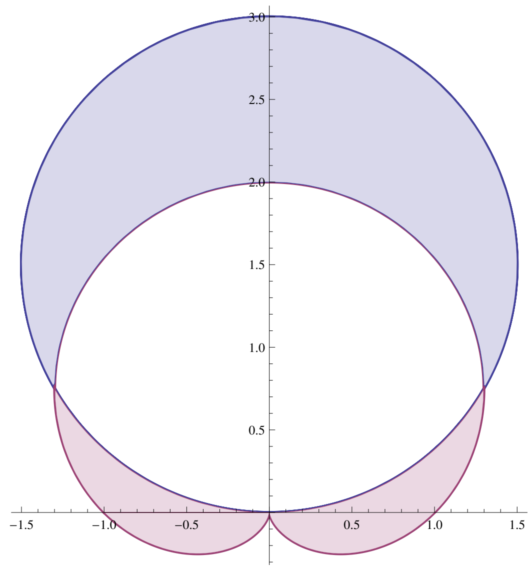
1 Questions:

2 Problems:

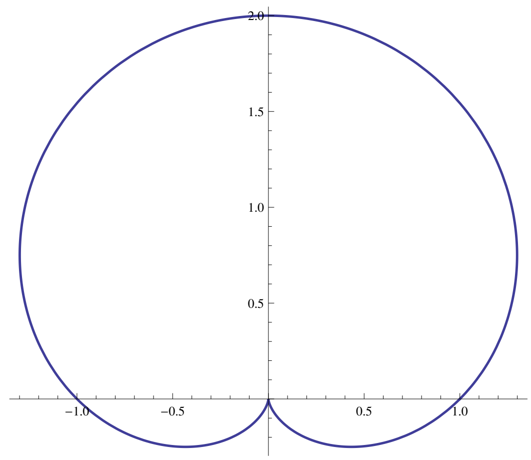
1. Convert the equation $x = 5$ to polar coordinates.
2. Sketch the curve $r = \cos(2\theta)$.
3. Find the area inside the curve $r = 3\sin(\theta)$ and outside $r = 1 + \sin(\theta)$.



4. Solution to 3. We first find intersection points. Set $3\sin(\theta) = 1+\sin(\theta)$ or $\sin(\theta) = 1/2$. $\theta = \pi/6, 5\pi/6$.

$$\begin{aligned}
A_B &= \frac{1}{2}\int_{\pi/6}^{5\pi/6} [9\sin^2(\theta) - (1+\sin(\theta))^2]d\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} [9\sin^2(\theta) - (1+2\sin(\theta)+\sin^2(\theta))]d\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} [8\sin^2(\theta) - 2\sin(\theta) - 1]d\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} [4(1-\cos(2\theta)) - 2\sin(\theta) - 1]d\theta \\
&= \frac{1}{2}\int_{\pi/6}^{5\pi/6} [3 - 4\cos(2\theta) - 2\sin(\theta)]d\theta \\
&= \frac{1}{2}[3\theta - 2\sin(2\theta) + 2\cos(\theta)]_{\pi/6}^{5\pi/6} \\
&= \frac{1}{2}[\frac{15\pi - 3\pi}{6} - 2\sin(5\pi/3) + 2\sin(\pi/3) + 2\cos(5\pi/6) - 2\cos(\pi/6)] \\
&= \frac{1}{2}[2\pi - \sqrt{3} + \sqrt{3} - \sqrt{3} - \sqrt{3}] = \frac{1}{2}(2\pi - 2\sqrt{3}) = \pi - \sqrt{3}.
\end{aligned}$$

5. Find the arc length of the polar curve $r = 1 + \sin(\theta)$



6. Solution to 5: $L = \int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta$: $\alpha = 0, \beta = 2\pi$. Need $\frac{dr}{d\theta}$.

$$\frac{dr}{d\theta} = \cos(\theta) \Rightarrow \left(\frac{dr}{d\theta}\right)^2 = \cos^2(\theta)$$

$$\begin{aligned}
L &= \int_{-\pi/2}^{\pi/2} \sqrt{(1+\sin(\theta))^2 + \cos^2(\theta)}\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \sqrt{1 + 2\sin(\theta) + \sin^2(\theta) + \cos^2(\theta)}\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \sqrt{2 + 2\sin(\theta)}\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \frac{\sqrt{2+2\sin(\theta)}\sqrt{2-2\sin(\theta)}}{\sqrt{2-2\sin(\theta)}}\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \frac{\sqrt{4 - 4\sin^2(\theta)}}{\sqrt{2-2\sin(\theta)}}\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \frac{2\cos(\theta)}{\sqrt{2-2\sin(\theta)}}\, d\theta \quad u = 2 - 2\sin(\theta) \Rightarrow du = -2\cos(\theta) \\
&= \int_4^0 \frac{-1}{\sqrt{u}}\, du \\
&= \int_0^4 u^{-1/2}\, du \\
&= 2\sqrt{u}|_0^4 = 8
\end{aligned}$$

7. Solve the differential equation: $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$ if $y(0) = 1$.

$$\int (1+y^2)dy = \int (1+x^2)\, dx$$

$$y + \frac{y^3}{3} = x + \frac{x^3}{3} + C$$
$$1 + \frac{1}{3} = C = \frac{4}{3}.$$

8. Solve the differential equation: $x\sin(y)\dfrac{dy}{dx} = \ln(x)\cos(y)$.