11/06/12

1 Examples

$$\sum_{n=1}^{\infty} \ln(\frac{n}{3n+1})$$

Test for divergence:

$$\begin{aligned}\lim_{n\to\infty} \ln(\frac{n}{3n+1}) &= \ln(\lim_{n\to\infty} \frac{n}{3n+1}) \\ &= \ln(1/3) \neq 0.\end{aligned}$$

Therefore the series diverges.

$$\sum_{n=2}^{\infty} \frac{1}{[\ln(\ln(n))]^{\ln(n)}}$$

2 Power Series

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

For $|x| < 1$.

3 Alternating Series: 10.3 #67

$$\sum_{n=1}^{\infty} \frac{1+(-1)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{1+(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

If we apply the alternating series test to the second term: we check $1/n$ is decreasing as $n \to \infty$ and $1/n \to 0$ as $n \to \infty$. So by the alternating series test, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges. But the harmonic diverges. Therefore the original series diverges.

4 10.3 # 52

$$\sum_{n=1}^{\infty} \frac{n-\cos(n)}{n^3}$$

$$\sum_{n=1}^{\infty} \frac{n - \cos(n)}{n^2} = \sum_{n=1}^{\infty} \frac{n}{n^3} - \sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$$
$$= \sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$$

Notice that the first series converges by the p-series with $p = 2 > 1$. The second term apply the test for absolute convergence:

$$\sum_{n=1}^{\infty} \left| \frac{\cos(n)}{n^3} \right| \leq \sum_{n=1}^{\infty} \frac{1}{n^3}$$

The series on the right converges by p-series with $p = 3 > 1$. Therefore by comparison we have $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$ is absolutely convergent. Therefore the original series must converge since both pieces converge.