Homework # 13 Solutions

May 10, 2010

Solution (5.4.3)**.** We note that

$$\frac{d}{dx}\left(\sin(x) - \frac{1}{3}\sin^3(x) + C\right) = \cos(x) - \sin^2(x)\cos(x) = \cos(x)\left(1 - \sin^2(x)\right)$$
$$= \cos(x)\cos^2(x) = \cos^3(x).$$

Thus $\int \cos^3(x)dx = \sin(x) - \frac{1}{3}\sin^3(x) + C$.

Solution (5.4.11)**.** Some basic algebra reveals

$$\int \frac{x^3 - 2\sqrt{x}}{x}dx = \int x^2 - 2x^{-1/2}dx = \frac{1}{3}x^3 - 4x^{1/2} + C$$

Solution (5.4.43)**.** Because $|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x \leq 0$, we have that

$$\int_{-1}^{2}(x - 2|x|)dx = \int_{-1}^{0}(x - 2|x|)dx + \int_{0}^{2}(x - 2|x|)dx$$
$$= \int_{-1}^{0}(x + 2x)dx + \int_{0}^{2}(x - 2x)dx$$
$$= \int_{-1}^{0} 3xdx + \int_{0}^{2} -xdx = \frac{3}{2}x^2\Big|_{-1}^{0} - \frac{1}{2}x^2\Big|_{0}^{2}$$
$$= \frac{3}{2}(0^2 - (-1)^2) - \frac{1}{2}(2^2 - (0)^2) = -\frac{3}{2} - \frac{4}{2} = -\frac{7}{2}.$$

Solution (5.4.59)**.** The acceleration a is given by $a(t) = t + 4$. Therefore, using the fact that

$$v(t) - v(0) = \int_0^t a(s)ds = \int_0^t s + 4ds = (\frac{1}{2}s^2 + 4s)\Big|_0^t = \frac{1}{2}t^2 + 4t$$

Then using the fact that $v(0) = 5$, we find

$$v(t) = \frac{1}{2}t^2 + 4t + 5.$$

The distance travelled Δx during the time interval $0 \leq t \leq 10$ is then

$$\Delta x = x(10) - x(0) = \int_0^{10} v(t)dt = \int_0^{10} \frac{1}{2}t^2 + 4t + 5dt = (\frac{1}{6}t^3 + 2t^2 + 5t\Big|_0^{10} = \frac{500}{3} + 200 + 50 = \frac{1250}{3}.$$

Solution (5.5.11)**.** Using $u = 2x + x^2$, we have that $du = 2(x+1)dx$, and therefore

$$\int (x+1)\sqrt{2x+x^2}dx = \frac{1}{2}\int \sqrt{u}dx = \frac{1}{2}\frac{2}{3}u^{3/2} + C$$
$$= \frac{1}{3}u^{3/2} + C = \frac{1}{3}(2x+x^2)^{3/2} + C$$

Solution (5.5.53)**.** Using $u = 1 + 2x^3$, we have that $du = 6x^2dx$. Moreover when $x = 0$, we have that $u = 1$ and when $x = 1$, we have that $u = 3$. Therefore

$$\int_0^1 x^2(1+2x^3)^5dx = \frac{1}{6}\int_1^3 u^5du = \frac{1}{36}u^6\Big|_1^3 = \frac{1}{36}(3^6 - 1^6) = 20.222.$$

Solution (5.5.57)**.** Noting that $\tan^3(-x) = -\tan^3(x)$, we have that $\tan^3(x)$ is an odd function. Integrating $\tan^3(x)$ over the symmetric domain $[-\pi/6, \pi/6]$ therefore gives us 0. That is,

$$\int_{-\pi/6}^{\pi/6} \tan^3(\theta)d\theta = 0.$$

Solution (5.5.69)**.** Let $u = e^z + z$. Then $du = (e^z + 1)dz$. Moreover, when $z = 0$, we have $u = 1$ and when $z = 1$, we have that $u = e + 1$. It folllows that

$$\int_0^1 \frac{e^z+1}{e^z+z}dz = \int_1^{e+1} \frac{1}{u}du = \ln|u||_1^{e+1}$$
$$= \ln(e+1) - \ln(1) = \ln(e+1).$$