MATH165 Homework 2. Solutions

February 2. 2010

Problem 1. 2.1.3 Point $P(1, \frac{1}{2})$ lies on $y = \frac{x}{1+x}$.

1. Q is the point $(x, \frac{x}{1+x})$.
   The slope of the secant PQ is given by

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

   where P is represented by (x_1, y_1) and Q is shown by (x_2, y_2).

   (a) $x = 0.5$. Then $Q = (0.5, 0.333333)$. Slope of $PQ = 0.333333$.

   (b) $x = 0.9$. Slope of $PQ = 0.263158$.

   (c) $x = 0.99$. Slope=0.251256.

   (d) $x = 0.999$. Slope=0.250125.

   (e) $x = 1.5$. Slope=0.2.

   (f) $x = 1.1$. Slope=0.238095.

   (g) $x = 1.01$. Slope=0.248756.

   (h) $x = 1.001$. Slope=0.249875.

2. The slope appears to be $\frac{1}{4}$.

3. The equation of line in slope point from is given by:

   $$y - y_1 = m(x - x_1)$$

where m is the slope, and (x_1, y_1) represents one point lying on the line. So, in this case, $m = \frac{1}{4}$ and $(x_1, y_1) = (1, \frac{1}{2})$.
$\Rightarrow y - \frac{1}{2} = \frac{1}{4}(x-1) \Rightarrow y = \frac{1}{4}x + \frac{1}{4}$.

Problem 2. 2.1.5

1. $y = y(t) = 40t - 16t^2$. At $t = 2$, $y = 40(2) - 16(2)^2 = 16$. Average velocity is given by $v_{ave} = \frac{y(2+h)-y(2)}{(2+h)-2} = -24 - 16h$, if $h \neq 0$.
   (a) $h = 0.5$. $v_{ave} = -32 ft/sec.$
   (b) $h = 0.1 . v_{ave} = -25.6 ft/sec.$
   (c) $h = 0.05 . v_{ave} = -24.8 ft/sec.$
   (d) $h = 0.01$. $v_{ave} = -24.16 ft/sec.$
2. Instantaneous velocity as $h \to 0$ is $-24 ft/sec.$

Problem 3. 2.2.4

1. $\lim_{x\to 0} f(x) = 3$.
2. $\lim_{x\to 3^-} f(x) = 4$.
3. $\lim_{x\to 3^+} f(x) = 2$.
4. Since, the righthand limit in part (3) $\neq$ lefthand limit in part(2), $\lim_{x\to 3} f(x)$ does not exist.
5. $f(3) = 3$.

Problem 4. 2.2.7

1. $\lim_{t\to 0^-} g(t) = -1$.

2. $\lim_{t\to 0^+} g(t) = -2$.
3. Since, the lefthand limit in part(1) $\neq$ righthand limit in part (2), $\lim_{t\to 0} g(t)$ does not exist.
4. $\lim_{t\to 2^-} g(t) = 2$.
5. $\lim_{t\to 2^+} g(t) = 0$.
6. Since, the lefthand limit in part (4) $\neq$ righthand limit in part (5), $\lim_{t\to 2} g(t)$ does not exist.
7. $g(2) = 1$.
8. $\lim_{t\to 4} g(t) = 3$.

Problem 5. 2.2.12 The function $f(x)$ is defined as

$$f(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-1)^2 & \text{if } x \geq 1 \end{cases}$$

Solution: The limit exists for all points except ± 1. Refer to figure 1.

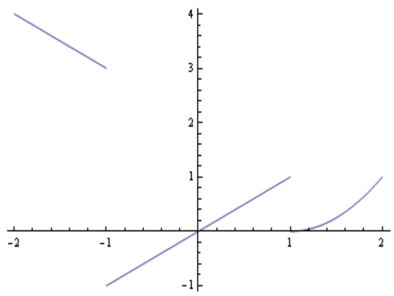

Figure 1: Graph of $f(x)$

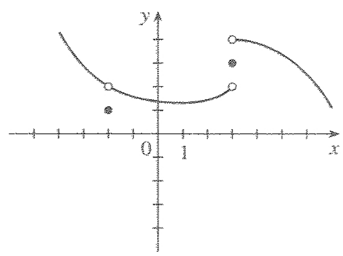

Figure 2: Graph for 2.2.15

Problem 6. 2.2.15 If $\lim_{x\to 3^+} f(x) = 4$,$\lim_{x\to 3^-} f(x) = 2$,$\lim_{x\to -2} f(x) = 2$,$f(3) = 3$,$f(-2) = 1$.
Refer to figure 2.

Problem 7. 2.2.19 Here, $f(x) = \frac{e^x-1-x}{x^2}$. And we are supposed to evaluate $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$.

(a) For $x = 1$, $f(x) = \frac{e^1-1-1}{1^2} = 0.718282$.

(b) For $x = -1$, $f(x) = 0.0.367879$.

(c) For $x = 0.1,\ f(x) = 0.517092.$

(d) For $x = -0.1,\ f(x) = 0.0.483472.$

(e) For $x = 0.5,\ f(x) = 0.594885.$

(f) For $x = -0.5, f(x) = 0.426123.$

(g) For $x = 0.01 f(x) = 0.501671.$

(h) For $x = -0.01 f(x) = 0.498337.$

So, $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^x-1-x}{x^2} = 0.5$

Problem 8. 2.2.21 Find $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$.

(a) For $x = 1,\ f(x) = \frac{\sqrt{5}-2}{1} = 0.236068$.Similarly, other computations can be done.

(b) For $x = -1, f(x) = 0.267949$

(c) For $x = 0.5 f(x) = 0.242641$

(d) For $x = -0.5 f(x) = 0.258343$

(e) For $x = 0.1,\ f(x) = 0.248457$

(f) For $x = -0.1, f(x) = 0.251582$

(g) For $x = 0.01 f(x) = 0.249844$

(h) For $x = -0.01 f(x) = 0.250156$

Problem 9. 2.2.27 Determine $\lim_{x\to 1} \frac{2-x}{(x-1)^2} = \frac{\lim_{x\to 1}(2-x)}{\lim_{x\to 1}(x-1)^2} = \frac{1}{0} = \infty.$

Problem 10. Estimate $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$.

(a) $x = -0.001$, then $f(x) = (1-0.001)^{\frac{1}{-0.001}} = 2.71964$. Similarly, other computations can be done.

(b) $x = -0.0001$, then $f(x) = 2.71842$.

(c) $x = -0.00001$ then $f(x) = 2.71830$.

(d) $x = -0.000001$ then $f(x) = 2.71828$.

(e) $x = 0.000001$ then $f(x) = 2.71828$

(f) $x = 0.00001$ then $f(x) = 2.71827$

(g) $x = 0.0001$ then $f(x) = 2.71815$

(h) $x = 0.001$ then $f(x) = 2.71692$

It appears that $\lim_{x\to 0}(1+x)^{\frac{1}{x}} \approx 2.71828$ which is exactly the value of e.