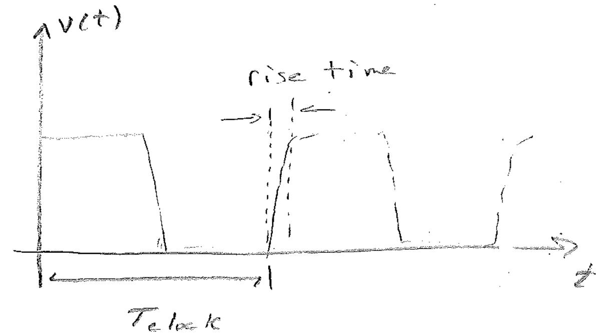
Time + Freq. domains in Signal Integrity

- Two main areas of interest: risetime + bandwidth
- These two areas will be investigated on interconnects.
- Talk about TLs.
- We first start w/ clock frequency.

$$F_{clock} = \frac{1}{T_{clock}}$$

or $$F = \frac{1}{T}$$



where F_{clock} (F) is the freq. of the clock + T_{clock} (T) is the period of the clock.

- Next, the rise time is related to how long it takes for the signal to transition from a low value to a high value. Two different defn. exist for rise time:

1) 10-90 rise time - time from 10% → 90%
2) 20-80 rise time - time from 20% → 80%.

∃ a similar defn. for fall time.

- Rise time is related to the turn on time of transistors attached (or loading) the interconnect

Sine waves in the freq. domain

There are four very useful properties of sine waves:

1) Any waveform in the time domain can be completely & uniquely described by combinations of sine waves.

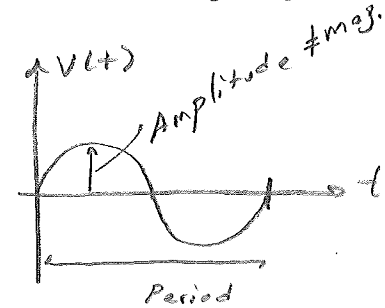
2) Any two sine waves w/ different freq. are orthogonal to each other.

3) They are well defined mathematically

4) They have a value everywhere w/ no infinities & they have derivatives that have no infinities anywhere.

- The following fully describes a sine wave

  - Frequency $(\omega = 2\pi f)$
  - Amplitude
  - Phase



The Fourier Transform

There are three types of FT:

- Fourier Integral (FI) - a $\pm\infty$ integral

- Discrete Fourier Transform (DFT)
- Fast Fourier Transform (FFT)

Discrete sampling of a signal & transformation.

Spectrum of an ideal square wave:

For an ideal square wave we have

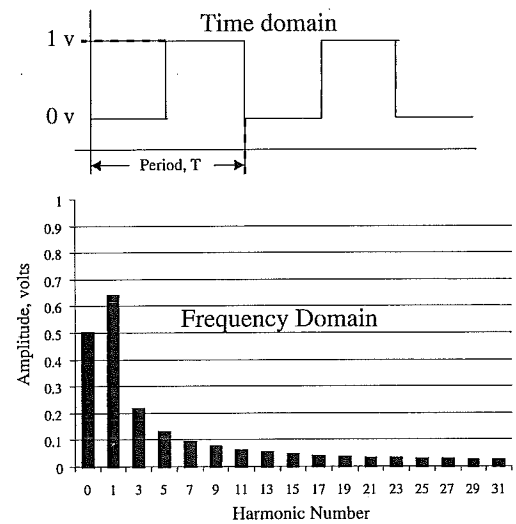


Figure 2-6 Time and frequency domain views of an ideal square wave.

We can compute the amplitude of the n^{th} freq. component of the 50% duty cycle square wave as: $A_n = \dfrac{2}{\pi n}$

where n is the odd harmonic number.

The Inverse Fourier Transform

- To construct a time-domain signal from the freq. domain we simply construct the associated sinusoid from each freq. comp. & sum the signals.

Consider the illustration.

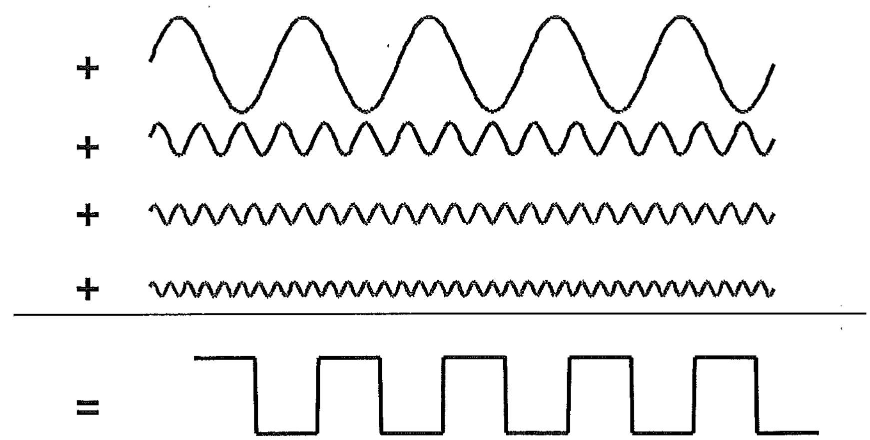


Figure 2-7 Convert the frequency-domain spectrum into the time-domain waveform by adding up each sine-wave component.

Bandwidth and rise time

The BW of a square wave is inversely proportional to the rise time.

Analytically, it can be shown that →.

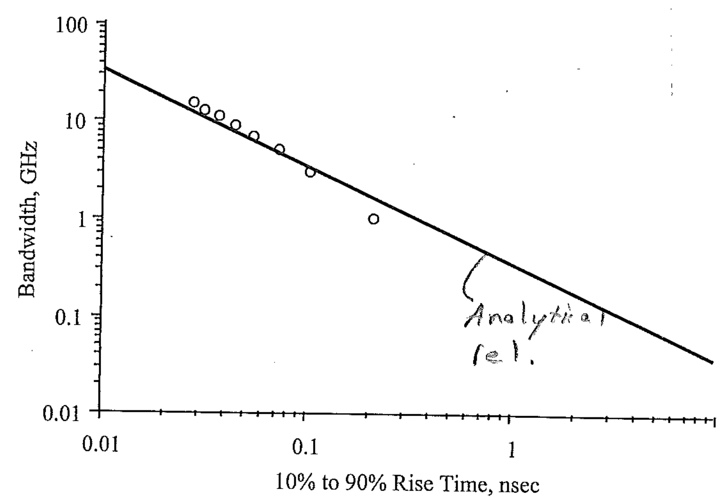


Figure 2-11 Empirical relationship between the bandwidth of a signal and its 10–90 rise time, as measured from a re-created ideal square wave with each harmonic added one at a time. Circles are the values extracted from the data; line is the approximation of BW = 0.35/rise time.

A good approx. is

$$BW = \frac{0.35}{RT}$$

where BW is in GHz

+ the 10-90 RT is in nS.

Ex: If RT = 1nS ⇒ BW = .35 GHz = 350 MHz.

or if BW = 3 GHz ⇒ RT = .1nS.

Bandwidth and clock frequency

- Signals w/ the same clock freq. can have different bandwidths. This is because the RT can be different between signals.

- To predict the BW of a clock directly from the freq. we have to assume a risetime. For this class we will assume that the RT is 7% of the clock period.

$\Rightarrow$ The period is ≈ 15 times $\left(1/.07 = 14.28\right)$ the RT.

We do know $BW = \frac{.35}{RT}$ + $T = 15 \cdot RT$

$$\Rightarrow BW = \frac{.35 \cdot 15}{T} = .35 \cdot 15 \cdot f \approx \boxed{5 \cdot f_{clock}}$$

A defn. for BW: To eval. the BW of a waveform, we are really asking what is the highest freq. component that is just barely above 70% of the same harmonic of an equiv. ideal squarewave.

Bandwidth of a model

When we refer to the BW of a model, we are referring to the highest sine-wave freq. component where the model will accurately predict the actual behavior of the structure it is represented.

Consider the following interconnect example:

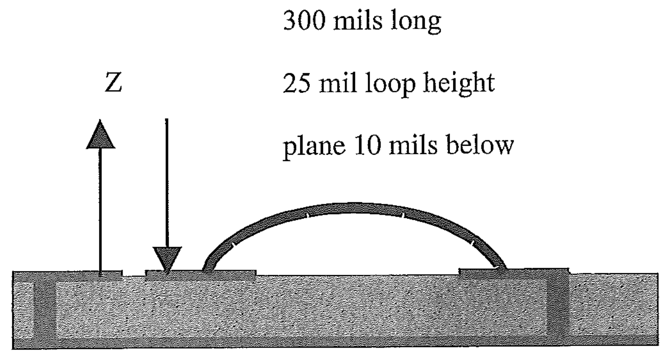


Figure 2-17 Diagram of a wire-bond loop between two pads, with a return path about 10 mils beneath the wire bond.

different models

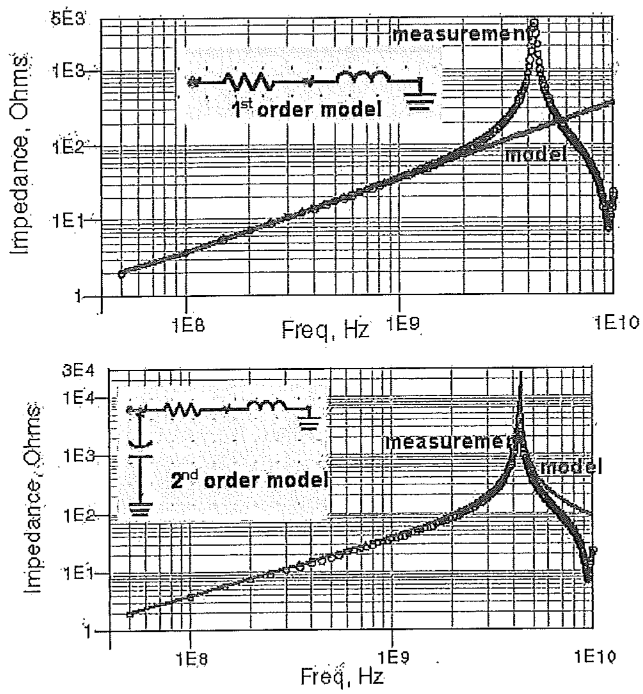


Figure 2-18 Top: Comparison of the measured impedance and the simulation based on the first-order model. The agreement is good up to a bandwidth of about 2 GHz. Bottom: Comparison of the measured impedance and the simulation based on the second-order model. The agreement is good up to a bandwidth of about 4 GHz. The bandwidth of the measurement is 10 GHz, measured with a GigaTest Labs Probe Station.

BW of an interconnect

- The BW of an interconnect refers to the -3dB cuttoff point.

Example

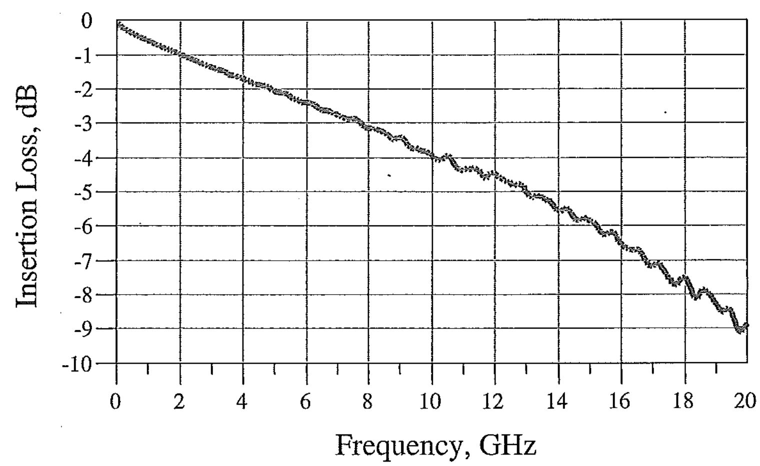


Figure 2-19 Measured transmitted amplitude of different sine-wave signals through a 4-inch-long transmission line made in FR4. The 3 dB bandwidth is seen to be about 8 GHz for this cross section and material properties. Measured with a GigaTest Labs Probe Station.

The BW of an interconnect is directly related to the minimum RT an interconnect can support. The RT exiting an interconnect can be approx. as $RT_{out}^2 = RT_{in}^2 + RT_{interconnect}^2$

where RT_{out} = 10-90 RT of the output sig.

RT_{in} = 10-90 RT of the input sig.

$RT_{interconnect}$ = 10-90 RT of the interconnect.

<u>Ex</u>: A 4in long interconnect has an input sig. w/ $RT_{in} = 50p\,sec$ + a $BW_{interconnect} = 8GHz$. $\Rightarrow RT_{intercon} = \frac{.35}{8} = .043ns$

$\Rightarrow RT_{out} = \sqrt{(50p\,sec)^2 + (.043s)^2} \approx \boxed{65.9ps}$

- As a rule-of-thumb, in order for the RT of the signal to be increased by the interconnect by less than 10%, the RT of the interconnect should be shorter than 50% of the RT of the signal.

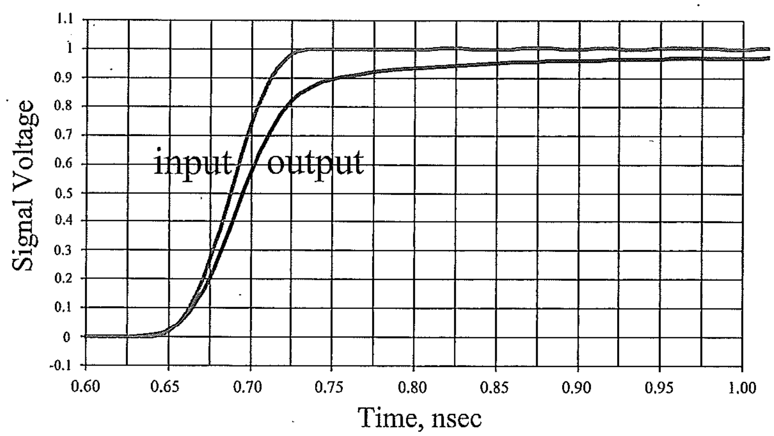


Figure 2-20 Measured input and transmitted signal through a 4-inch long, 50-Ohm transmission line in FR4 showing the rise-time degradation. The input rise time is 50 psec. The predicted output rise time is 67 psec based on the measured bandwidth of the interconnect. Measured with a GigaTest Labs Probe Station.

In the freq. domain, to support the transmission of a 1GHz BW signal, we want the interconn. to have a BW of atleast 2GHz.

Impedance & Electrical Models

- See attached sheets -

1.0.1. Non-ideal capacitors

If current flows through a real conductor, then we are guaranteed to have inductance and resistance. If we have more than one conductor at different potentials, we are guaranteed to have capacitance. With this in mind, we will investigate non-ideal components. The first is the capacitor. The impedance of a non-ideal capacitor is shown in Fig. 1. We can see that the "real" (non-ideal) capacitor behaves similar to an ideal capacitor up to a resonant point. Then the inductance of the component begins to dominant. This behavior leads to the equivalent circuit in Fig. 2. If R_2 is neglected, then $Z = R + j(\omega L - 1/(\omega C))$. This then gives $f_o = \frac{1}{2\pi\sqrt{LC}}$.

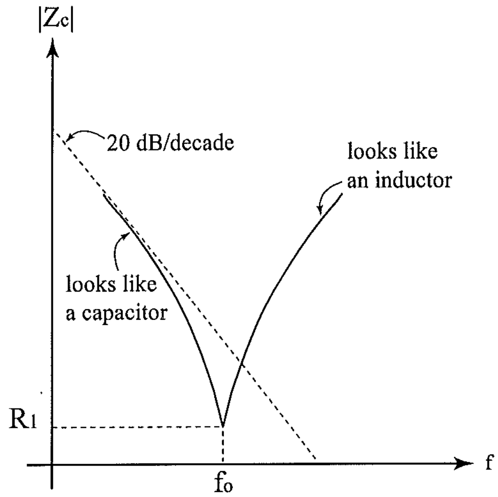


Figure 1. Impedance of a non-ideal capacitor.

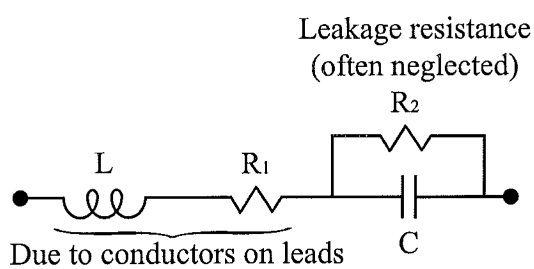
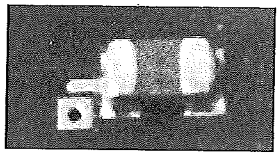


Figure 2. Equivalent circuit of a non-ideal capacitor.

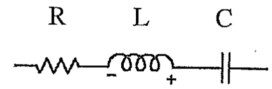
Consider:

Real physical component



Equivalent electrical circuit model of ideal circuit elements:

Topology:



Parasitic values:

C = 0.67 nF

R = 0.50 Ω

L = 1.78 nH

Figure 3-3 The two worldviews of a component, in this case a 1206 decoupling capacitor mounted to a circuit board and an equivalent circuit model composed of combinations of ideal circuit elements.

1st + 2nd order model.

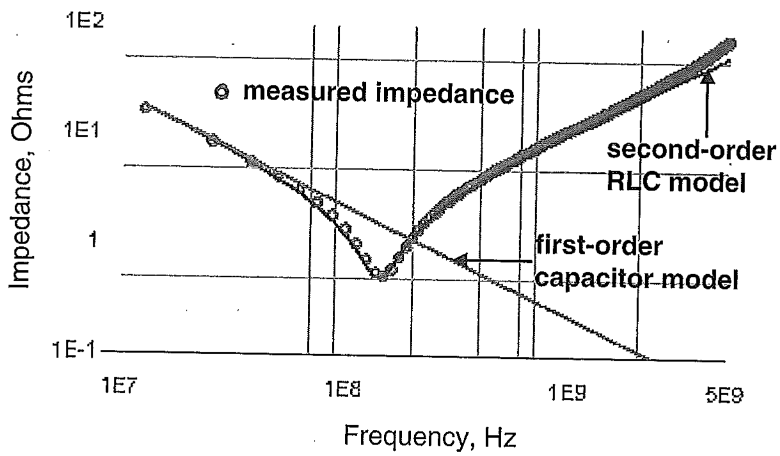


Figure 3-8 Comparison of the measured impedance of a real decoupling capacitor and the predicted impedance of a simple first-order model using a single C element and a second-order model using an RLC circuit model. Measured with a GigaTest Labs Probe Station.

1.0.2. Non-ideal resistors

There are three different types of resistors:

1) Composition - the inductance is very small.

2) Film type - the inductance is moderate - has some effect.

3) Wirewound - the inductance is very significant but can dissipate several Watts of power.

The impedance of a typical non-ideal resistor is shown in Fig. 3. The equivalent circuit is shown in Fig. 4.

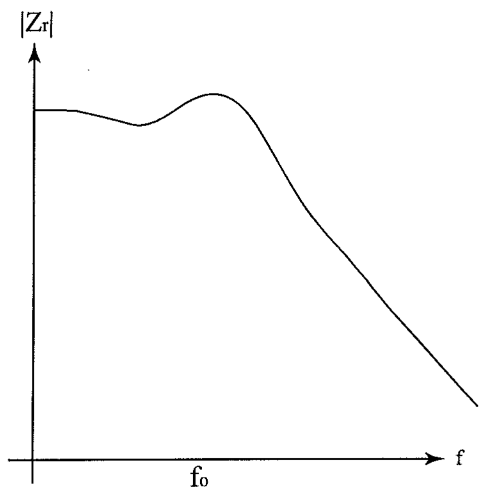


Figure 3. Impedance of a non-ideal resistor.

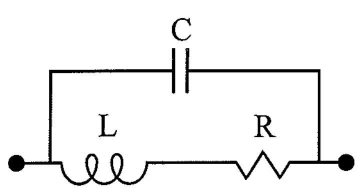


Figure 4. Equivalent circuit of a non-ideal resistor.

1.0.3. Non-ideal inductors

There are four types of inductors:

1) Wound on an open core (Fig. 5 a)):

a) permeable core - iron, etc.

b) non-permeable material as core (wood, air, .etc)

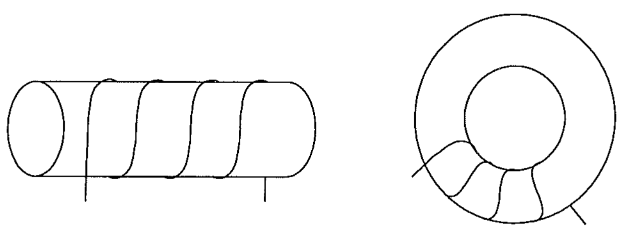
2) Wound on a closed core (Fig. 5 b)):

a) permeable core - iron, etc.

b) non-permeable material as core (wood, air, .etc)

We summarize the behavior of open- and closed-core inductors in the following table (1 is the worst and 4 is the best):

AIR	cause of radiated emissions	susceptible to radiated fields
open core	1	2
close core	3	4
MAGNETIC		
open core	2	1
closed core	4	3



a) b)

Figure 5. a) Open core inductor and b) closed core inductor.

One other aspect to consider is don't pass to much current such that the inductor saturates. The impedance of a typical non-ideal inductor is shown in Fig. 6. The equivalent circuit is shown in Fig. 7.

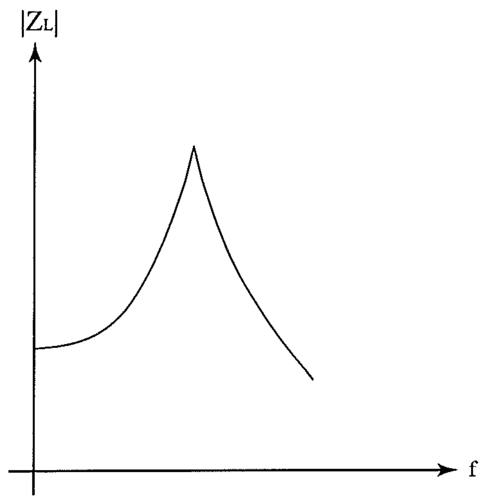


Figure 6. Impedance of a non-ideal inductor.

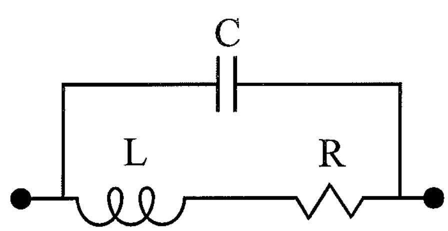


Figure 7. Equivalent circuit of a non-ideal inductor.

Finally, consider the modeling of a finite TL:

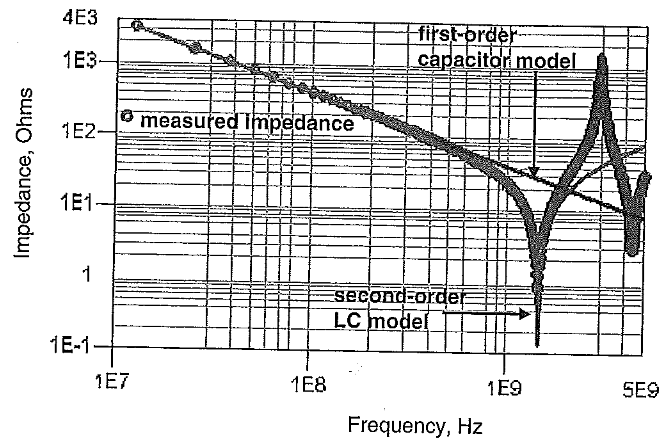
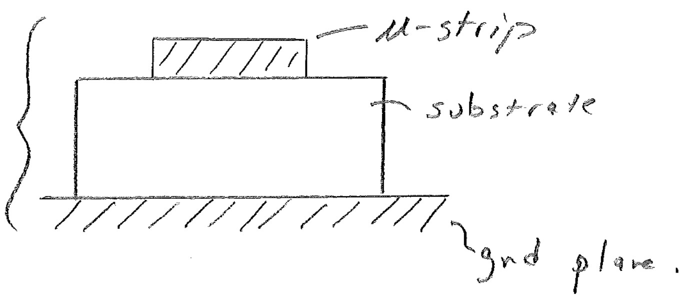


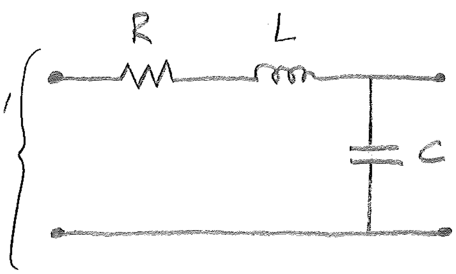
Figure 3-14 Measured impedance of a 1-inch-long microstrip trace and the simulated impedance of a first- and second-order model. The first-order model is a single C element and has a bandwidth of about 1 GHz. The second-order model uses a series LC circuit and has a bandwidth of about 2 GHz.

<u>Resistance</u>

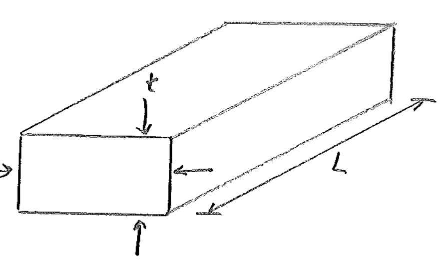
Physical world



Electrical world



The cross section of a conductor has the following notation:



$R = \rho \frac{L}{A}$ where

A = cross-sectional area cm^2.

R = resistance

ρ = bulk resistivity Ω-cm

L = length - cm

Bulk Resistivity

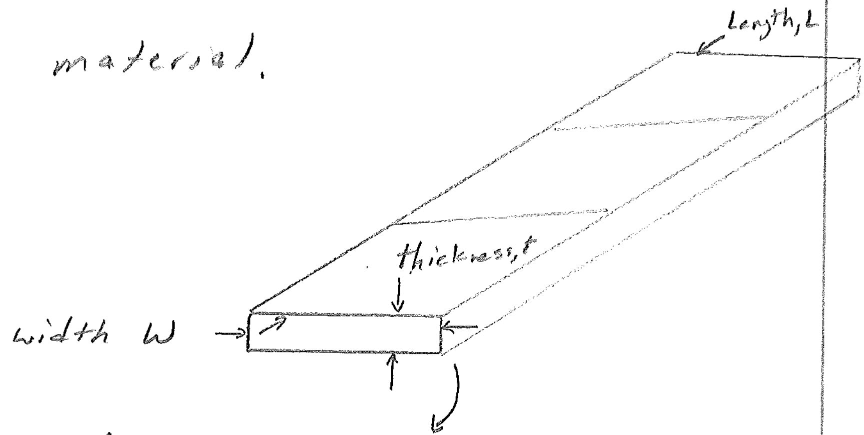
Bulk resistivity is a fundamental property that all conductors have & the units are Ω-in or Ω-cm.

- Bulk resistivity is not a property of the structure or object made from a material, it is a property of the material.

- We also have $\rho = \frac{1}{\sigma}$ where σ is the conductivity of the material.

Sheet Resistance

Consider: →



A uniform sheet cut into a number of squares $n = d/w$.

If the trace width is uniform, then

$$R = \rho \frac{d}{t \times w} = \left(\frac{\rho}{t}\right)\left(\frac{d}{w}\right)$$

$$= R_{sq} \cdot \frac{d}{w}$$

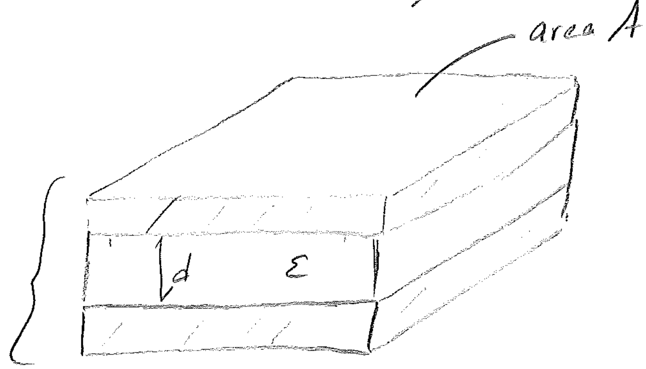
where R_{sq} is denoted as the sheet resistance.

$\Rightarrow \quad R = R_{sq} \cdot n$

Capacitance

- The capacitance between any two conductors is a measure of their capacity to store charge at the cost of voltage or $C = Q/V$.

Capacitance between two parallel plates



It can be shown that $\boxed{C = \frac{\varepsilon A}{d}}$.

The dielectric constant

- permittivity is strictly a material property.

$$\varepsilon_r = \frac{C}{C_0}$$

where ε_r = relative dielectric constant of the material
C = Cap. w/ the plates completely surrounded by the material

C_0 = Cap. w/ surrounding air.

Power + Ground Planes + Decoupling capacitance

- used to reduce Voltage rail collapse.

If the pwr. dissipation of the chip is P, the time until the voltage droop increases to 5% of the supply voltage, b/c of the decoupling cap. is:

$$\delta_t = C \cdot .05 \cdot \frac{V^2}{P}$$

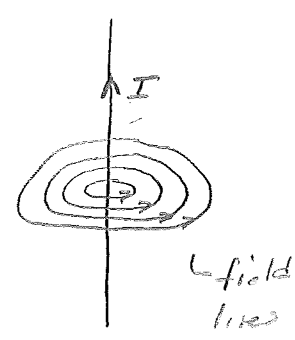
δ_t = time in seconds until droop exceeds 5%.
C = decoupling cap. (F)
.05 = 5% voltage droop.
P = Avg. pwr. dissipated on the chip (W).
V = supply voltage (V).

Inductance

A practical approach to inductance depends on three fundamental principles:

Principle #1: There are circular rings of Magnetic field lines around ALL currents.

- The field lines are always complete circles + always enclose some current. There must be some current encircled by the field lines rings.

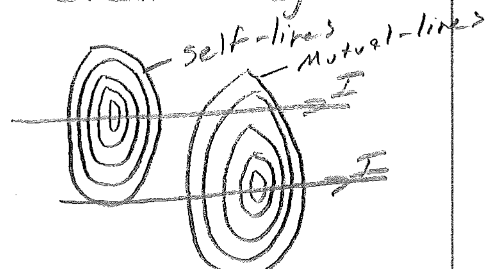


- Since we are dealing w/ H-fields, diel. materials do not interact w/ the H-fields around a current I in any manner. ⇒ The field lines do not change due to a diel. material.

Principle #2: Inductance is the # of Webers of field line rings around a conductor per Amp of current through it or $L = \Psi / I$.

- Note, inductance is about the number of rings of magnetic-field lines enclosing a current, not about the absolute value of the magnetic field at any one point.

- Also, inductance is a measure of the efficiency of the conductors to create magnetic field line rings.



Self- + Mutual Inductance

- We use the term self-inductance to refer to the number of field line rings around a wire, per Amp of current in its own wire. When we say inductance we refer to self-inductance.
- We define mutual-inductance to refer to the number of field lines around one wire, per Amp of current in another wire.

Principle #3: When the # of field line rings around a conductor changes, there will be a voltage induced across the ends of the conductor.

↳ This is why we care about inductance.

The voltage created is directly related to how fast the total field-line rings change:

$$V = \frac{\Delta N}{\Delta t}$$

where V = induced votage across the ends of the conductor.

ΔN = the # of field line rings that change.

Δt = the time in which they change.

- Induced voltage is the fundamental reason why inductance plays such an important role in signal integrity.

- When another conductor supports a current that changes, we typically have a noise voltage on a different conductor where

$$V_{noise} = M \frac{dI}{dt}$$

Noise voltage ↗ Mutual Inductance ↑ current on the 2nd wire.

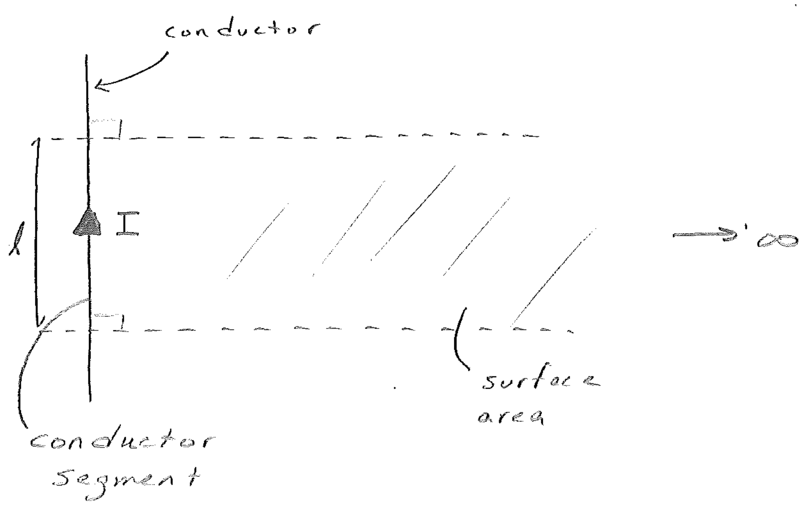
Partial Inductance and Ground Bounce

When we are looking at the field lines from only a segment of a loop current & neglecting the remaining currents in the loop, this inductance is called the partial inductance. When considering partial inductance, we are ignoring the other currents in the loop. However, this concept is very useful for understanding ground bounce.

- In practice, we cannot measure partial inductance. This is because an isolated current cannot exist & must always exist in a loop.

Consider:

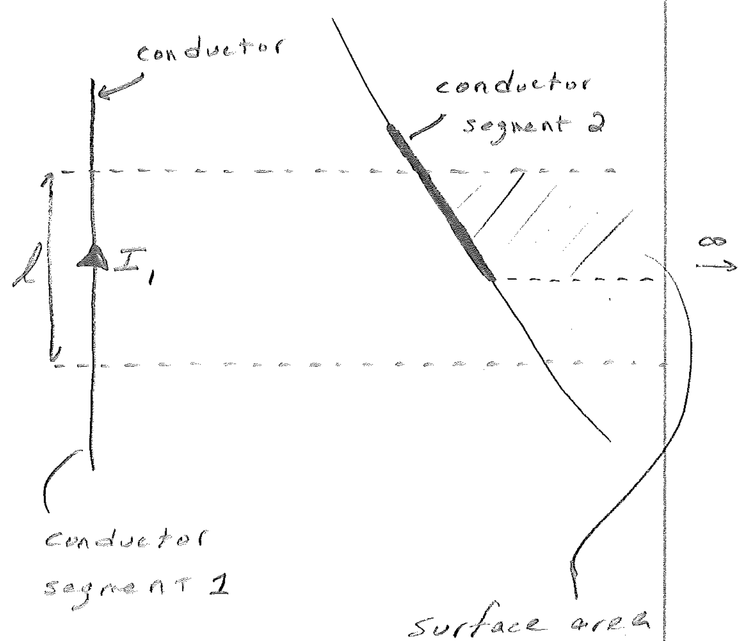
$L = \frac{\Phi}{I}$ - The total Φ is summed over the infinite surface area.



∴ the partial self-inductance of a conductor segment is the magnetic flux penetrating the surface area between the conductor segment + infinity, divided by the current in the conductor segment.

- We also have <u>partial</u> <u>mutual</u> <u>inductance</u>

The partial mutual inductance between ~~the~~ two conductor segments is the ratio of the magnetic flux penetrating the surface area between the 2nd conductor segment + infinity, divided by the current I_1 in the first conductor segment.



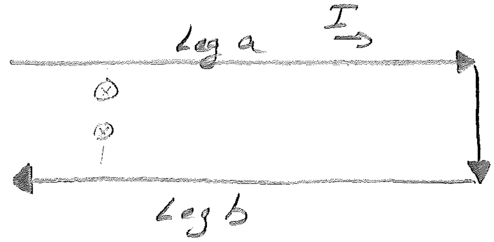
The partial self- + mutual- inductance of a round wire of radius r + length d can be approximated as:

$$L = 5d\left[\ln\left(\frac{2d}{r}\right) - 3/4\right] \quad + \quad M = 5d\left[\ln\left(\frac{2d}{s}\right) - 1\right]$$

spacing.

Inductance & ground bounce

consider:



We can compute the effective inductance of one leg, based on the partial inductances of two legs. The partial inductance of both legs can be written as L_a & L_b, & the partial mutual inductance can be labeled as L_{ab}.

Around Leg b we have $N_b = I \times L_b$ field lines from its own current & $N_{ab} = L_{ab} \times I$ mutual field lines from the current on Leg a.

$\Rightarrow N_{total} = N_b - N_{ab} = (L_b - L_{ab}) \times I$

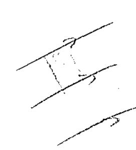
where N_{total} is the total # of field lines around leg b.

We denote $(L_b - L_{ab})$ the total or effective inductance of leg b.

- When the second leg is the return path, and the current in the loop changes, $L_b - L_{ab}$ will determine how much voltage is generated across leg b. This generated voltage is called <u>ground bounce</u> and is denoted as:

$$V_{gb} = L_{total} \times \frac{dI}{dt} = (L_b - L_{ab}) \times \frac{dI}{dt}.$$

total L of just the return path



- To decrease the partial self-inductance of the leg means making the return path as short & wide as possible w/ return planes. To increase Lab, decrease the distance between a signal trace + its return.

- Rules for via placement for decoupling capacitors:

- Keep the center-to-center spacing between vias of the same current direction at least as far apart as the length of the via + keep the center-to-center spacing between vias w/ opposite direction current much closer than the length of the vias to increase Lab.

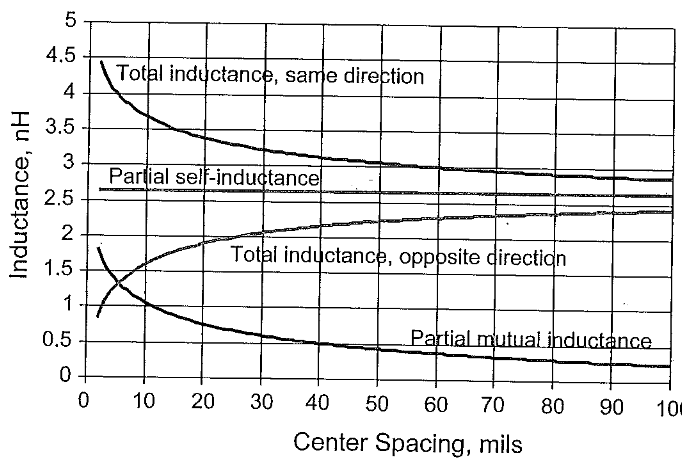


Figure 6-10 Total inductance of one 100-mil-long wire bond when an adjacent wire bond carries the same current, for the two cases of the currents in the same direction and in opposite directions. The wires are pulled apart, comparing the total inductances with the partial self- and mutual inductances.

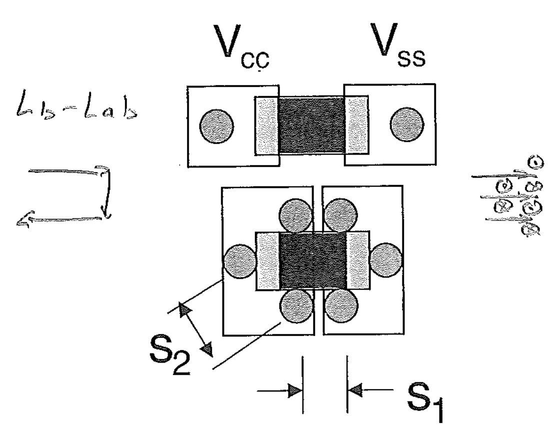
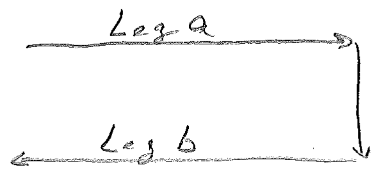


Figure 6-11 Via placement for decoupling capacitor pads between Vcc and Vss planes. Top: conventional placement. Bottom: optimized for low total inductance and lowest voltage-collapse noise: s_2 > via length, s_1 < via length.

Loop Inductance

- when we measure the inductance of the entire current loop, we call it <u>loop inductance</u>.

Next, the total inductance of the loop can be written as:



$$L_{Loop} = L_a - L_{ab} + L_b - L_{ab}$$

$$= L_a + L_b - 2L_{ab}.$$

where L_a is the partial inductance of leg a,

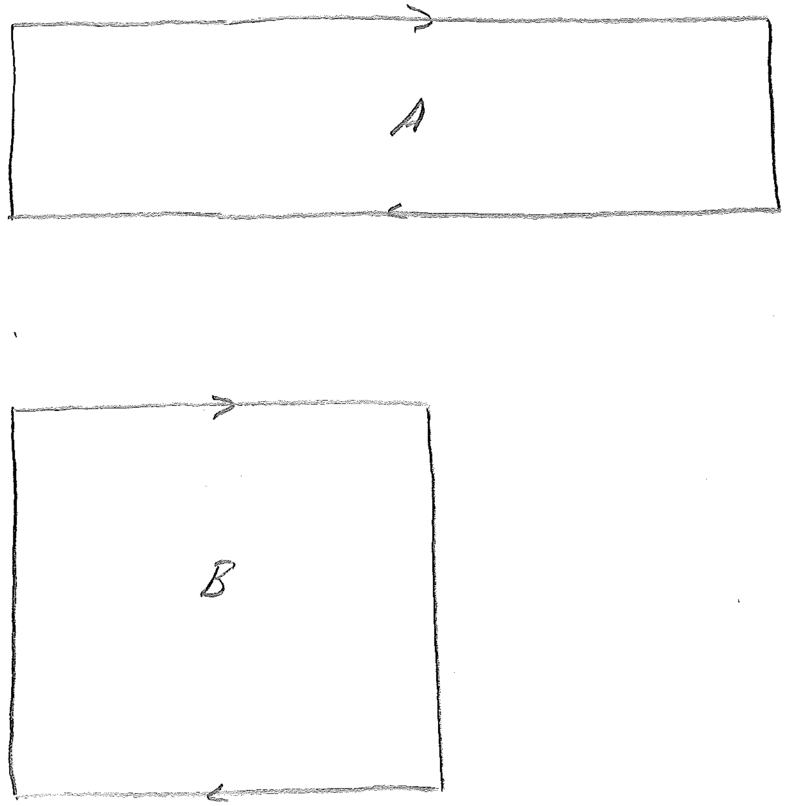
L_b is the " " " leg b

L_{ab} " " " mutual inductance between legs a + b.

The L_{Loop} eqn. says that if we bring the two legs closer together, L_{ab} will increase.

$\Rightarrow L_{Loop}$ will decrease.

Next, consider the two loops.



same area but loop B has less partial mutual inductance & hence more loop inductance.

For a circular loop,

$$L_{Loop} = 32 \cdot R \cdot \ln\left[\frac{4R}{D}\right] \text{nH}$$

where R is the radius of the loop in inches
D is the diameter of the wire in inches

Also, the loop inductance of two adjacent, straight-round wires, assuming one carries the return current is given by:

$$L_{Loop} = 10 \cdot length \cdot \ln\left[\frac{s}{r}\right] \; nH$$

where length is the length of the wires in inches

s - is the center-to-center separation of the wires in inches

r - is the radius of the rods in mils.

Power - Distribution Network (PDN) and Loop Inductance

The power + ground paths are called the power distribution network (PDN). The PDN is under the power integrity heading or subject.

- The role of the PDN is to deliver a constant voltage across the pwr. + grd. pads of each chip.

- Between a pwr. src. (regulator) & a load exists the impedance of the PDN. As the load changes states, the pwr. requirements change, which results in a changing need for current. ⇒ the current pulled from the PDN changes & b/c of the PDN impedance, the PDN introduces a voltage change (called a voltage droop or rail collapse) between the src. & load. ∴ to minimize voltage droop or rail collapse, the design goal is to keep the impedance of the PDN below a certain value.
- One way to do this is to add decoupling capacitance to the power loop. This decoupling cap. can provide a local charge for a small amount of time until the PDN can respond to the change in load.

To compute the needed decoupling cap.

we start w/ $C = \frac{Q}{V}$.

$$\Rightarrow \Delta V = \frac{\Delta Q}{C}$$

where ΔV is the change in voltage across the cap.

ΔQ is the charge depletion from the cap.

Next, to find out how much charge is needed, a good estimate is to use the avg. pwr. of the load (chip): $P = VI = I = \frac{P}{V}$

Since $I = \frac{dq(t)}{dt}$, $I = \frac{\Delta Q}{\Delta t} = \frac{P}{V} = \frac{\Delta V C}{\Delta t}$

$$\Rightarrow \Delta t = \frac{\Delta V \cdot C \cdot V}{P} = \frac{\overset{5\%\text{ droop}}{.05} \cdot V \cdot C \cdot V}{P} = \frac{.05 \cdot C \cdot V^2}{P}$$

or the capacitance required to decouple for a given amount of time:

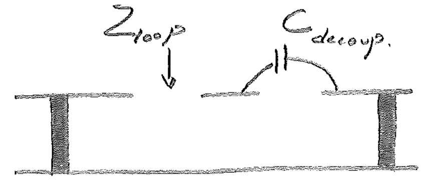
$$\boxed{C = \underbrace{\frac{1}{0.05}}_{\text{Voltage droop allowed}} \cdot \frac{P}{\underbrace{V^2}_{\text{rail voltage}}} \cdot \Delta t}$$

where Δt is the time charge is flowing from the cap.

P is the pwr. dissipated on the chip.

- When implementing a decoupling cap. we need to consider the loop inductance.

The decoup. cap. + Loop inductance is in series.



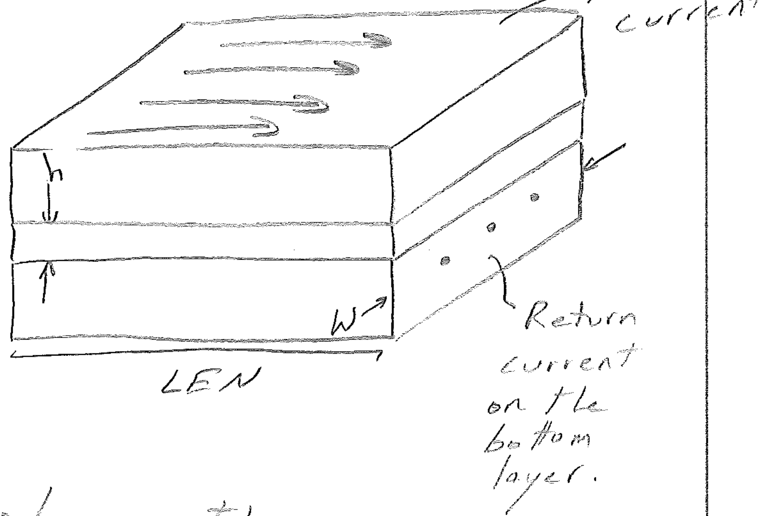
$\Rightarrow \exists$ a resonance + for freq. above this resonance, the impedance increases only because of L_{Loop}.

- $\therefore$ the only way to decrease the impedance of a decoupling cap. at high freq. is to decrease its loop self-inductance.

- Here are a few notes to help reduce L_{loop}:
  1) Keep vias short by assigning pwr. + gnd. planes close to the surface
  2) Use small body size cap.
  3) Use very short conn. between the cap. pads + the vias to the underlying plane.
  4) Use mult. cap. in parallel.

Loop Inductance per Square of Planes

Consider the following planes carrying a surface current.



- The larger (wider) the planes, the lower the partial self-inductance.
- The longer the planes, the larger to partial self-inductance.
- The closer the planes, the larger the mutual inductance.

⇒ wide + close planes lower the loop inductance.

- In the case where the width is much larger than the spacing, or $W \gg h$,

$$L_{Loop} = \mu_0 h \frac{LEN}{W} \text{ (nH)}.$$

$\mu_0 = 32 pH/mil$

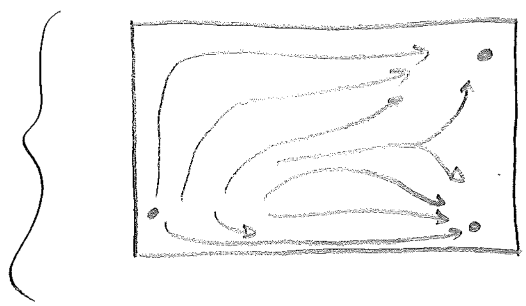
h = spacing in mils.

- By spacing the gnd. + pwr. planes as close together as possible will decrease loop inductance, decrease rail collapse, decrease gnd. bounce + decrease EMI.

Loop Inductance of Planes + Via Contacts

Actually, on a plane we have via connections + a non-uniform current dist.

To solve for loop inductance in this case, a 3D field solver is required.

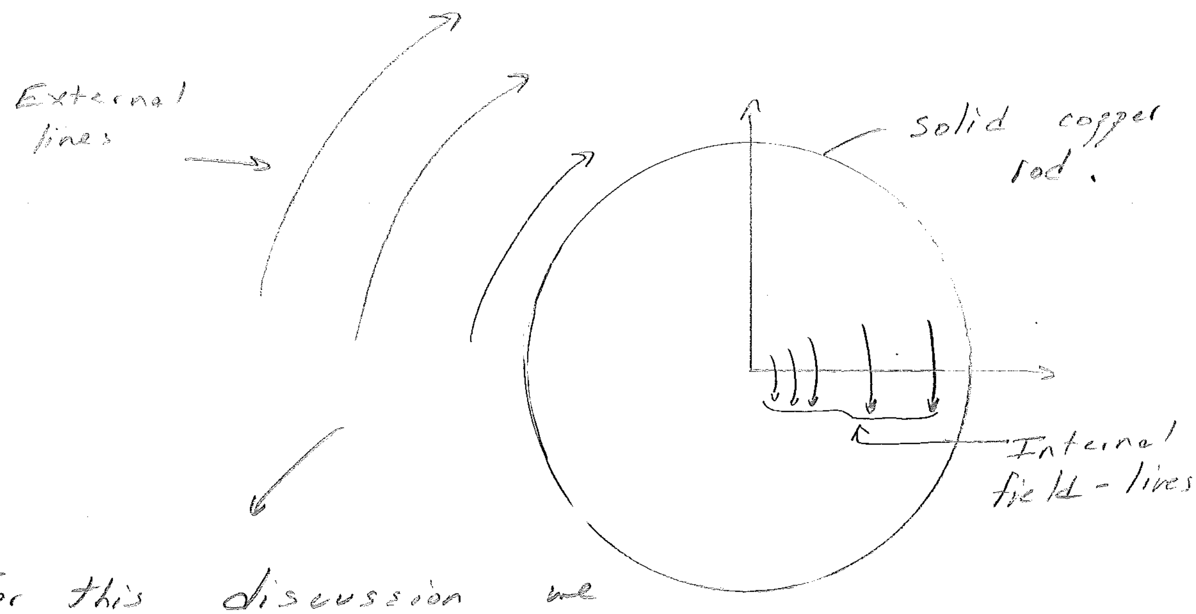


Current distributions + Skin Depth

- up to this point, we have been assuming that the current in the conductors were uniformly distributed throughout the conductors.

- This is not the case for high-freq. currents.

- To understand this we need to consider the field lines in the conductor.



For this discussion we will seperate the self-inductance of the wire to internal self-inductance & external self-inductance.

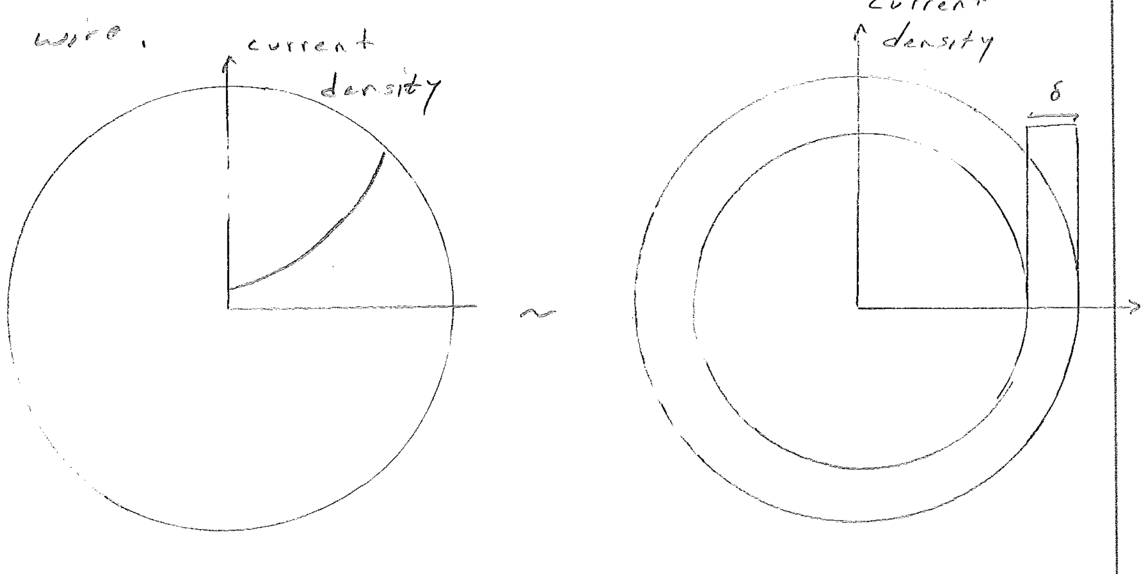
- The internal field lines see the conductor & are affected by the metal.
- The internal field lines in the wire always enclose a current.
- ∴ currents closer to the center of the wire will be enclosed by more field-

lines. ⇒ the currents closer to the center have a higher self-inductance than the currents closer to the outside.

- Since an AC current travels along the path w/ least impedance, more current travels down the low impedance paths which are further away from the center of the conductor.

- As the freq. increases, the impedance of the self-inductance near the center of the wire increases, thus forcing more current to travel down the outer surface of the wire.

consider:



- The current near the outer edges of the wire can be approx as a current shell w/ thickness δ + uniform current density where:

$$\delta = \sqrt{\frac{1}{\sigma \pi \mu_0 \mu_r f}}$$

δ = skin depth in m.

σ = conductivity in siemens/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

μ_r = relative permeability

f = freq. in Hz.

For copper w/ $\sigma = 5.6 \times 10^7$ S/m + $\mu_r \doteq 1$,

$$\boxed{\delta \approx 66 \text{ microns} \sqrt{1/f}}$$

where δ is in microns + f is in MHz.

- For example, the skindepth of a 10MHz signal is 20.8 microns. ⇒ on a 1oz copper trace, which a thickness of 34 microns, the current will not travel through the entire conductor.

- Next, because more current is traveling near the outer surface of the conductor, a high freq (or freq. dependent) resistance will be observed where

$$R_{HF} = \frac{\rho}{W\delta}$$

R_{HF} = HF Resistance
ρ = bulk resistivity of copper
W = line width of the signal trace
δ = skin depth of copper at the HF.

OR: $$\frac{R_{HF}}{R_{DC}} = \frac{t}{\delta}$$

where $R_{DC} = \frac{\rho}{Wt}$ + t is the thickness of the trace.

<u>Eddy Currents</u>

- The induced currents on a conductor are called <u>eddy currents</u>.

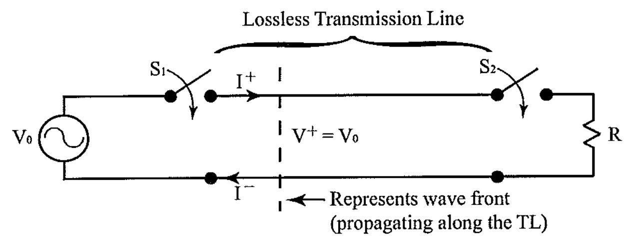
- Eddy currents can have a large effect on a ckt. if they are induced on a plane.

CHAPTER 1. TRANSMISSION LINES

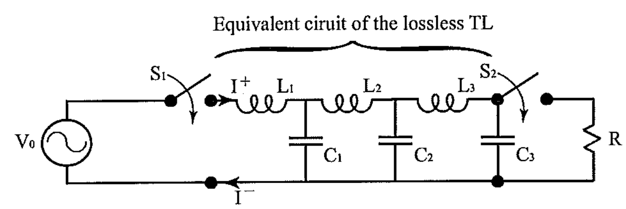
Transmission lines (TL) are used to deliver power from a source to a load. This can be done by using coaxial lines or conductors made out of wire. When distances are large enough between the source and the load, we use TL theory to find the voltage (V) and current (I) along the TL-line. These TLs are usually categorized as electrically small or electrically large structures. This then results in the following two methods to analyze the propagation of a wave along a TL:

Lumped Elements - if the time delay between the source and load is negligible ($L << \lambda/10$).

Distributed Elements - if the time delay between the source and load is not negligible ($L \gtrsim \lambda/10$).



a)



b)

Figure 1. a) Propagation of a wave along a TL; the equivalent circuit of the lossless TL.

1.1. Transmission Line Propagation

First, consider the lossless TL shown in Fig. 1 a). If switch S_1 is closed, a wave travels down the TL. If switch S_2 is closed right before the wave arrives at the load R, a wave will be reflected back towards the source. The amount reflected back depends on R (or if S_2 is open or closed). The equivalent circuit for the TL in Fig. 1 a) is shown in Fig. 1 b). When S_1 is closed, the current in L_1 begins to build. This then charges C_1. C_1 then begins to supply current to L_2 as it approaches max voltage...etc. This then leads to a wave propagating down the TL towards the load R.

1.1.1. Transmission Line Equations

Next, consider the per-unit equivalent (lumped element model) circuit of a short TL shown in Fig. 2. R represents the conductor loss, L represents the line inductance, C represents the line capacitance and G represents the loss in the dielectric between the conductors. Now we want to derive expressions for $V(z)$ and $I(z)$ on the TL shown in Fig. 2 in terms of R, L, C and G.

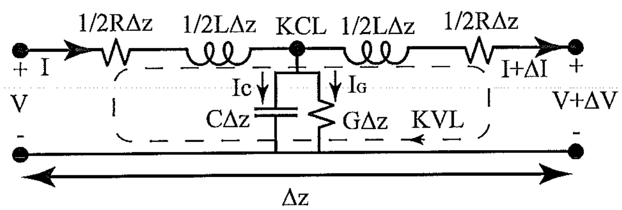


Figure 2. Per-unit equivalent circuit of a TL.

First, using KVL on the circuit shown in Fig. 2:

$$V = \frac{1}{2}R\Delta z I + \frac{1}{2}L\Delta z\frac{\partial I}{\partial t} + \frac{1}{2}L\Delta z\frac{\partial}{\partial t}(I+\Delta I) + \frac{1}{2}R\Delta z(I+\Delta I) + V + \Delta V. \tag{1.1}$$

$\Rightarrow$

$$\frac{V}{\Delta z} = \frac{1}{2}RI + \frac{1}{2}L\frac{\partial I}{\partial t} + \frac{1}{2}L\left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t}\right) + \frac{1}{2}R(I+\Delta I) + \frac{V}{\Delta z} + \frac{\Delta V}{\Delta z}. \tag{1.2}$$

$\Rightarrow$

$$\frac{\Delta V}{\Delta z} = -RI - L\frac{\partial I}{\partial t} - \frac{1}{2}R\Delta I - \frac{1}{2}L\frac{\partial \Delta I}{\partial t}. \tag{1.3}$$

Next, evaluating the limit $\Delta z \to 0$ (i.e., $\lim_{\Delta z\to 0}$) we have $I + \Delta I \to I$, $\Delta I \to 0$ and $\frac{\Delta V}{\Delta z} \to \frac{\partial V}{\partial z}$. This then reduces 1.3 to the following:

$$\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t}\right). \tag{1.4}$$

Next, using KCL on the circuit shown in Fig. 2:

$$\begin{aligned} I &= I_G + I_C + I + \Delta I \\ &= G\Delta z\left(V + \frac{\Delta V}{2}\right) + C\Delta z\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2}\right) + I + \Delta I. \end{aligned} \tag{1.5}$$

$\Rightarrow$

$$\frac{I}{\Delta z} = G\left(V + \frac{\Delta V}{2}\right) + C\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2}\right) + \frac{I}{\Delta z} + \frac{\Delta I}{\Delta z}. \tag{1.6}$$

$\Rightarrow$

$$\frac{\Delta I}{\Delta z} = -G\left(V + \frac{\Delta V}{2}\right) - C\frac{\partial}{\partial t}\left(V + \frac{\Delta V}{2}\right). \tag{1.7}$$

Next, evaluating the limit $\Delta z \to 0$ (i.e., $\lim_{\Delta z \to 0}$) we have $V + \Delta V \to I$, $\Delta V \to 0$ and $\frac{\Delta I}{\Delta z} \to \frac{\partial I}{\partial z}$. This then reduces 1.7 to the following:

$$\frac{\partial I}{\partial z} = -\left(GV + C\frac{\partial V}{\partial t}\right). \tag{1.8}$$

Equations 1.4 and 1.8 are referred to as the telegraphist's equations. Their solutions lead to the wave equations on the TL. Next, differentiating (1.4) w.r.t. z and (1.8) w.r.t. t we get:

$$\frac{\partial^2 V}{\partial z^2} = -R\frac{\partial I}{\partial z} - L\frac{\partial^2 I}{\partial t \partial z} \tag{1.9}$$

and

$$\frac{\partial^2 I}{\partial z \partial t} = -G\frac{\partial V}{\partial t} - C\frac{\partial^2 V}{\partial t^2}. \tag{1.10}$$

Next, substituting (1.8) and (1.10) into (1.9) gives

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV. \tag{1.11}$$

Similar substitutions result in the following expression:

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2} + (LG + RC)\frac{\partial I}{\partial t} + RGI. \tag{1.12}$$

Equations (1.11) and (1.12) represent the general wave equations for the TL in Fig. 2.

1.2. Lossless Propagation

1.2.1. Wave velocity and characteristic impedance

For lossless propagation we have $R = G = 0$ in Fig. 2. This then simplifies (1.11) and (1.12) to the following:

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \tag{1.13}$$

and

$$\frac{\partial^2 I}{\partial z^2} = LC\frac{\partial^2 I}{\partial t^2}. \tag{1.14}$$

Solving the second order partial differential equations in (1.13) and (1.14) results in the following assumed solutions:

$$V(z,t) = f_1(t - z/\nu) + f_2(t + z/\nu) = V^+ + V^- \tag{1.15}$$

where ν represents the wave velocity, V^+ represents the forward traveling wave and V^- represents the backward traveling wave. The $t - z/\nu$ represents the forward traveling wave. As t increases, z must also increase to sustain $f(0)$ (the wave front). To solve for the wave velocity we consider the solution of $V(z,t)$ to $\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}$. With out loss of generality we consider only f_1.

$\Rightarrow$

$$\frac{\partial V(z,t)}{\partial z} = \frac{\partial f_1}{\partial z} = \frac{\partial f_1}{\partial(t - z/\nu)}\frac{\partial(t - z/\nu)}{\partial z} = \frac{-1}{v}f_1' \tag{1.16}$$

where f_1' is the partial derivative w.r.t. the argument. Similarly we have

$$\frac{\partial V(z,t)}{\partial t} = \frac{\partial f_1}{\partial t} = \frac{\partial f_1}{\partial(t - z/\nu)}\frac{\partial(t - z/\nu)}{\partial t} = f_1', \tag{1.17}$$

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{\nu^2}f_1'' \tag{1.18}$$

and

$$\frac{\partial^2 f_1}{\partial t^2} = \frac{1}{\nu^2} f_1''. \tag{1.19}$$

Notice that (1.18) and (1.19) are simply derivatives of f_1 w.r.t. z and t, respectively. Substituting (1.18) and (1.19) into (1.13) results in the following expression:

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{\nu^2} f_1'' = LC f_1'' = LC \frac{\partial^2 V}{\partial t^2} \tag{1.20}$$

or

$$\frac{1}{\nu^2} f_1'' = LC f_1''. \tag{1.21}$$

Canceling f_1'' results in the following expression for the *wave velocity* ν:

$$\nu = \frac{1}{\sqrt{LC}}. \tag{1.22}$$

Similarly for (1.14) we have the following solution:

$$I(z,t) = \frac{1}{L\nu}\Big[f_1(t - z/\nu) - f_2(t + z/\nu)\Big] = I^+ + I^- = \frac{1}{Z_0} V(z,t) \tag{1.23}$$

where

$$Z_0 = \sqrt{\frac{L}{C}}. \tag{1.24}$$

Z_0 is referred to as the *characteristic impedance* of the TL. Notice the expressions for the wave velocity and characteristic impedance are written entirely in terms of L and C. This indicates that ν and Z_0 are dependent only on the dimensions of the physical structure and not time and frequency.

1.2.2. Phase constant, phase velocity and wavelength

In this section we derive the expressions for the voltage and current along the TL when a steady-state sinusoidal source is used to drive the TL. Start by defining $f_1 = f_2 = V_0 \cos(\omega t + \phi)$. From the previous section we have $t = t \pm z/\nu_p$ where ν_p is referred to as the *phase velocity.*

$\Rightarrow$

$$\begin{aligned} V(z,t) &= |V_0|\cos[\omega(t+z/\nu_p)+\phi] \\ &= |V_0|\cos[\omega t \pm \beta z + \phi]. \end{aligned} \tag{1.25}$$

$\Rightarrow$

$$V^+ = V_f(z,t) = |V_0|\cos[\omega t + \beta z + \phi] \tag{1.26}$$

and

$$V^- = V_b(z,t) = |V_0|\cos[\omega t - \beta z + \phi] \tag{1.27}$$

where

$$\beta = \frac{\omega}{\nu_p}. \tag{1.28}$$

β is called the *phase constant* of the TL. If $t = 0$ (i.e., fix time and look at the spatial variation) then we have $V_f(z,0) = |V_0|cos[\beta z]$. $V_f(z,0)$ represents a periodic function that repeats w.r.t. a value of z. Denote this value as λ and call it the *wavelength* of the wave. This then gives $\beta\lambda = 2\pi$.

$\Rightarrow$

$$\lambda = \frac{2\pi}{\beta} = \frac{\nu_p}{f}. \tag{1.29}$$

Now, if $z = 0$ (fix position and look at time variation) then we have $V(0,t) = |V_0|\cos[\omega t]$. This is illustrated in Fig. 3. Notice that the sinusoid repeats every 2π.

$\Rightarrow$

$$T = \frac{1}{F} \tag{1.30}$$

where T is the period of the sinusoid.

1.2.3. Voltage and current along the transmission line

Here we want to represent the voltage along the TL as complex functions. Using Euler's identity we have $e^{\pm jx} = \cos(x) \pm j\sin(x)$.

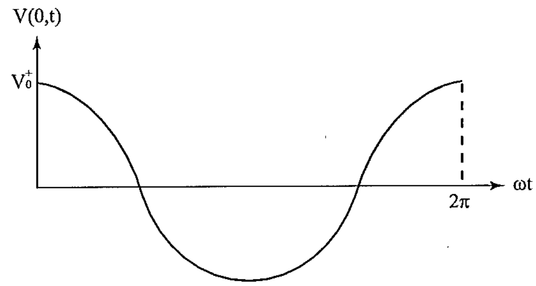


Figure 3. Sinusoidal voltage along the TL.

$\Rightarrow$

$$\cos(x) = Re\left[e^{\pm jx}\right] = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \tag{1.31}$$

and

$$\sin(x) = \pm Im\left[e^{\pm jx}\right] = \pm\frac{1}{2j}\left[e^{jx} - e^{-jx}\right] \tag{1.32}$$

$\Rightarrow$

$$\begin{aligned} V(z,t) &= |V_0|\cos[\omega t \pm \beta z + \phi] \\ &= \frac{1}{2}|V_0|\left[e^{j(\omega t \pm \beta z + \phi)} + e^{-j(\omega t \pm \beta z + \phi)}\right] \\ &= \frac{1}{2}|V_0|\left[e^{j\phi} + e^{-j\phi}\right]\left[e^{j(\omega t \pm \beta z)} + e^{-j(\omega t \pm \beta z)}\right] \\ &= \frac{1}{2}V_0\left[e^{j(\omega t \pm \beta z)} + e^{-j(\omega t \pm \beta z)}\right]. \end{aligned} \tag{1.33}$$

Note that V_0 was used to represent the complex voltage magnitude $\frac{1}{2}|V_0|\left[e^{j\phi} + e^{-j\phi}\right]$ in (1.33). Next, define the following

$$V_c(z,t) = V_0 e^{\pm j\beta z} e^{j\omega t} \tag{1.34}$$

and

$$V_s(z) = V_0 e^{\pm j\beta z} \tag{1.35}$$

where $V_s(z,t)$ is the complex instantaneous voltage and $V_s(z)$ is the phasor voltage. Again, from the general wave equation (1.11) we have

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} + (LG + RC)\frac{\partial V}{\partial t} + RGV. \tag{1.36}$$

In the phasor domain we also have the relation $\frac{\partial}{\partial t} \Leftrightarrow j\omega$.

$\Rightarrow$

$$\frac{d^2 V}{dz^2} = -\omega^2 LCV_s + j\omega(LG + RC)V_s + RGV_s. \tag{1.37}$$

$\Rightarrow$

$$\frac{d^2 V}{dz^2} = (R + j\omega L)(G + j\omega C)V_s = \gamma^2 V_s \tag{1.38}$$

where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$ is the *propagation constant* along the TL. Now assume that $V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$ for a solution to $\frac{d^2 V_s}{dz^2} = \gamma^2 V_s$. Similarly, assume $I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$. Substituting the assumed voltage and current solutions into the transformed expressions of (1.4) and (1.8) gives the following expressions:

$$\frac{dV_s}{dz} = -(R + j\omega L)I_s \tag{1.39}$$

and

$$\frac{dI_s}{dz} = -(G + j\omega C)V_s. \tag{1.40}$$

$\Rightarrow -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{-\gamma z} = -Z\left[I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right]$ where $Z_0 = R + j\omega L$. Equating the coefficients of $e^{-\gamma z}$ and $e^{\gamma z}$ gives $-\gamma V_0^+ = -ZI_0^+$ and $\gamma V_0^- = -ZI_0^-$. Therefore, the characteristic impedance

Z_0 is

$$\begin{aligned} Z_0 &= \frac{V_0^+}{I_0^+} \\ &= -\frac{V_0^-}{I_0^-} \\ &= \frac{Z}{\gamma} \\ &= \frac{Z}{\sqrt{ZY}} \\ &= \sqrt{\frac{Z}{Y}} \end{aligned} \tag{1.41}$$

where $Y = G + j\omega C$.

$\Rightarrow$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = |Z_0|e^{j\theta}. \tag{1.42}$$

1.3. Examples: Transmission Lines

1.3.1. Example 1

Consider an 80 cm long lossless TL with a source connected to one end operating at 600 MHz. The lumped element values of the TL are $L = 0.25\mu H/m$ and $C = 100pF/m$. Find Z_0, β and ν_p.

Solution: Using (1.42) we have

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0 + j\omega.25 \times 10^{-3}}{0 + j\omega 100 \times 10^{-12}}} = 50\Omega.$$

Also from (1.38) we have $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(0 + j\omega L)(0 + j\omega C)} = 0 + j\beta = j\omega\sqrt{LC}$.

$\Rightarrow \beta = 2\pi(600 \times 10^6)\sqrt{LC} = 18.85 rad/m$. Finally, we have $\nu_p = \omega/\beta = 2 \times 10^8 m/s$.

1.3.2. Example 2

A transmission line constructed of two parallel wires in air has a conductance of $G = 0\Omega$. The two parallel wires are made of good conductors, therefore it is assumed that $R = 0\Omega$. If $Z_0 = 50\Omega$, $\beta = 20$ rad/m and $f = 700$ MHz, find the per-unit inductance and capacitance of the TL.

Solution:

Since $R = G = 0$, $\beta = \omega\sqrt{LC} = 40 = 2\pi 700MHz\sqrt{LC}$ and $Z_0 = \sqrt{\frac{L}{C}} = 50$. Then the ratio of β and Z_0 results in the following:

$$\frac{\beta}{Z_0} = \omega C.$$

Solving for C then gives:

$$\begin{aligned} C &= \frac{\beta}{\omega Z_0} \\ &= \frac{20}{2\pi * 700MHz * 50} \\ &= 90.9pF/m. \end{aligned}$$

Finally, solving for L gives:

$$\begin{aligned} L &= Z_0^2 C \\ &= 50^2 * 90.9 \times 10^{-12} \\ &= 227nH/m. \end{aligned}$$

1.4. Wave Reflections — Talk about Ringing.

1.4.1. Reflection coefficient and transmission coefficient

Next, expressions for describing the reflections of wave along the TL are derived. To do this, consider the TL load in Fig. 4. Next, denote

$$V_i(z) = V_{0i}e^{-\alpha z}e^{-j\beta z} \tag{1.43}$$

and

$$V_r(z) = V_{0r}e^{+\alpha z}e^{+j\beta z}. \tag{1.44}$$

$\Rightarrow$

$$V_L(0) = V_i(z) + V_r(z) = V_{0i} + V_{0r} \tag{1.45}$$

and

$$I_L(0) = I_{0i} + I_{0r} = \frac{1}{Z_0}[V_{0i} - V_{0r}]. \tag{1.46}$$

Now define the *reflection coefficient* at the load as

$$\Gamma = \frac{V_{0r}}{V_{0i}} = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\phi_\Gamma}. \tag{1.47}$$

Now using Γ we can write the voltage at the load in terms of the reflection coefficient and the incident wave in the following manner:

$$V_L = V_{0i} + \Gamma V_{0i}. \tag{1.48}$$

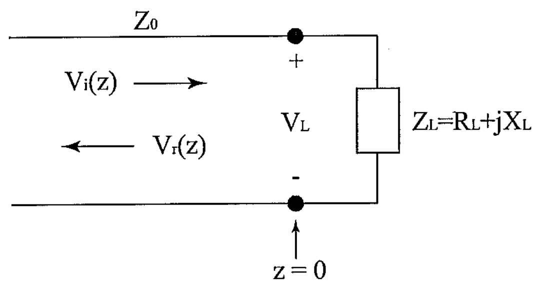


Figure 4. Voltage reflection at the load of a TL.

Now define the *transmission coefficient* as

$$\tau = \frac{V_L}{V_{0i}} = 1 + \Gamma = \frac{2Z_L}{Z_0 + Z_L} = |\tau| e^{j\phi_t}. \tag{1.49}$$

1.4.2. Reflected power

The time averaged power can be written as

$$< P >= \frac{1}{2} Re(V_s I_s^*) = \frac{1}{2} Re\left(V_0 e^{-\alpha z} e^{-j\beta z} \frac{V_0^*}{|Z_0| e^{j\theta}} e^{-\alpha z} e^{+j\beta z} \right) = \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} \cos\theta. \tag{1.50}$$

Note that θ in (1.50) refers to the angle on the characteristic impedance Z_0. Then for $z = L$ we have the following expression for the incident power:

$$< P_i >= \frac{1}{2}\frac{|V_0|^2}{|Z_0|}e^{-2\alpha L}\cos\theta. \tag{1.51}$$

To find the reflected power substitute in the reflected wave in (1.48):

$$\begin{aligned} < P_r > &= \frac{1}{2}Re\left(\Gamma V_0 e^{-\alpha z}e^{-j\beta L}\frac{(\Gamma V_0)^*}{Z_0 e^{j\theta}}e^{-\alpha z}e^{j\beta L}\right) \\ &= \frac{1}{2}\frac{|\Gamma|^2|V_0|^2}{|Z_0|}e^{-2\alpha L}\cos\theta. \end{aligned} \tag{1.52}$$

This then leads to the following expression for the *reflected power*

$$\frac{< P_r >}{< P_i >} = \Gamma\Gamma^* = |\Gamma|^2. \tag{1.53}$$

Similarly, for the *transmitted power*:

$$\frac{< P_t >}{< P_i >} = 1 - |\Gamma|^2. \tag{1.54}$$

1.5. Examples: Reflection Along the Transmission Line

1.5.1. Example 1

a) $Z_L = Z_0 = 50\Omega \Rightarrow \Gamma = 0$ and $\tau = 1$.

b) $Z_L = 0$, $Z_0 = 50\Omega \Rightarrow \Gamma = -1$ and $\tau = 0$.

c) $Z_L \to \infty$ (open), $Z_0 = 50\Omega \Rightarrow \Gamma = 1$ and $\tau = 2$.

Notice from the previous derivations and examples that we have $-1 \leq \Gamma \leq 1$ ($R_L \geq 0$) and $0 \leq \tau \leq 2$ ($R_L \geq 0$).

1.5.2. Example 2

A TL with $Z_0 = 100\Omega$ is loaded with a series connected 50 Ω resistor and a 10 pF capacitor. Find the reflection coefficient at the load at 100 MHz.

Solution:

The load impedance at 100 MHz is $Z_L = 50 - j159\Omega$. Using (1.47) gives

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j159 - 100}{50 - j159 + 100} = 0.762\angle - 60.78.$$

1.5.3. Example 3

Show that $|\Gamma| = 1$ for a purely reactive load.

Solution:

For this example let $Z_L = 0 + jX_L$. From (1.47) the reflection coefficient at the load is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX_L - Z_0}{jX_L + Z_0} = \frac{-(Z_0 - jX_L)}{Z_0 + jX_L} = \frac{-\sqrt{Z_0^2 + X_L^2}e^{-j\theta}}{\sqrt{Z_0^2 + X_L^2}e^{j\theta}} = -e^{-j2\theta}.$$

This then gives $|\Gamma| = |-e^{-j2\theta}| = 1$.

1.6. The Coaxial Transmission Line

1.6.1. High frequency analysis

A cross-section of a coaxial TL is shown in Fig. 5. The radius of the inner conductor is a, the radius of the inner wall on the outer conductor is b and the radius of the outer wall on the outer conductor is c. It can be shown that the per unit values of the coaxial TL are:

$$C = \frac{2\pi\varepsilon'}{\ln(\frac{b}{a})} \quad \left(\frac{F}{m}\right), \tag{1.55}$$

$$G = \frac{2\pi\sigma}{\ln(\frac{b}{a})} \quad \left(\frac{S}{m}\right), \tag{1.56}$$

$$L_{ext} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) \quad \left(\frac{H}{m}\right), \tag{1.57}$$

$$R = \frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a} + \frac{1}{b}\right) \quad \left(\frac{\Omega}{m}\right), \tag{1.58}$$

and

$$Z_0 = \sqrt{\frac{L_{ext}}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon'}}\ln\left(\frac{b}{a}\right) \quad (\Omega) \tag{1.59}$$

where σ is the conductivity of the TL, ε is the permittivity of the TL and μ is the permeability of the TL.

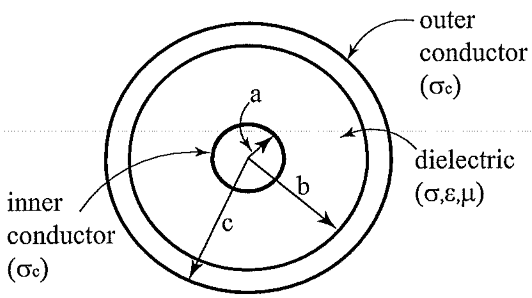


Figure 5. Cross-section of a coaxial TL.

1.6.2. Low frequency analysis

Also for the LF analysis of the coaxial TL we have the following expressions:

$$C = \frac{2\pi\varepsilon'}{\ln(\frac{b}{a})} \quad \left(\frac{F}{m}\right), \tag{1.60}$$

$$G = \frac{2\pi\sigma}{\ln(\frac{b}{a})} \quad \left(\frac{S}{m}\right), \tag{1.61}$$

$$R = \frac{1}{\pi\sigma_c}\left(\frac{1}{a^2} + \frac{1}{c^2 - b^2}\right), \tag{1.62}$$

and

$$L = \frac{\mu}{2\pi}\left[\ln\left(\frac{b}{a}\right) + \frac{1}{4} + \frac{1}{4(c^2+b^2)}\left(b^2 - 3c^2 + \frac{4c^4}{c^2 - b^2}\ln\left(\frac{c}{b}\right)\right)\right]. \tag{1.63}$$

1.7. Two-wire Transmission Line

1.7.1. High frequency analysis

A cross section of the two-wire TL is shown in Fig. 6. The radius of each wire is denoted as

a and the center of each wire is separated by a distance d. It is assumed that the two wires are emersed in a material with properties (σ, μ, ε'). It can be shown that the per unit values of the two-wire TL are:

$$C = \frac{\pi\varepsilon'}{\ln(\frac{d}{a})} \quad \left(\frac{F}{m}\right), \tag{1.64}$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(\frac{d}{2a})} \quad \left(\frac{S}{m}\right), \tag{1.65}$$

$$L = \frac{\mu}{\pi}\ln\left(\frac{d}{a}\right) \quad \left(\frac{H}{m}\right), \tag{1.66}$$

$$R = \frac{1}{\pi a \delta \sigma_c} \tag{1.67}$$

and

$$Z_0 = \sqrt{\frac{L}{C}}. \tag{1.68}$$

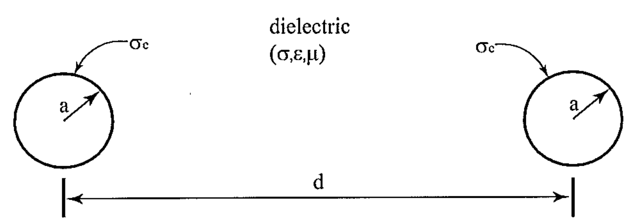


Figure 6. Cross-section of a two-wire TL.

1.7.2. Low frequency analysis

Also for the LF analysis of the two-wire TL we have the following expressions:

$$C = \frac{\pi\varepsilon'}{\cosh^{-1}(\frac{d}{2a})} \quad \left(\frac{F}{m}\right), \tag{1.69}$$

$$G = \frac{\pi\sigma}{\cosh^{-1}(\frac{d}{2a})} \quad \left(\frac{S}{m}\right), \tag{1.70}$$

$$L = \frac{\mu}{\pi}\left[\frac{1}{4} + \cosh^{-1}\left(\frac{d}{2a}\right)\right] \quad \left(\frac{H}{m}\right), \tag{1.71}$$

and

$$R = \frac{2}{\pi a^2 \sigma_c}. \tag{1.72}$$

1.8. Voltage Standing Wave Ratio (VSWR)

For the voltage standing wave ratio (VSWR) we start with the following:

$$\begin{aligned} V_{sT}(z) &= V_0 e^{-j\beta z} + \Gamma V_0 e^{j\beta z} \\ &= V_0\left[e^{-j\beta z} + |\Gamma| e^{j(\beta z+\phi)}\right] \\ &= V_0(1-|\Gamma|)e^{-j\beta z} + 2V_0|\Gamma| e^{j\phi/2}\cos(\beta z + \phi/2) \end{aligned} \tag{1.73}$$

where ϕ is the angle on the reflection coefficient. Converting (6.17) to the time domain we get:

$$\begin{aligned} V(z,t) &= Re\left[V_{sT}(z)e^{j\omega t}\right] \\ &= V_0(1-|\Gamma|)\cos(\omega t - \beta z) \\ &\quad +2V_0|\Gamma|\cos(\beta z + \phi/2)\cos(\omega t + \phi/2). \end{aligned} \tag{1.74}$$

The first term in the last expression in (6.18) describes the traveling wave and the second term in the last expression describes the standing wave along the TL. An example of a standing wave is illustrated in Fig. 7. The oscillations of the standing wave in Fig. 7 are described by the trigonometric terms in (6.18). We can also derive expressions for the position of the voltage maximums and minimums along the TL. The position of the voltage maximum is denoted as z_{max}

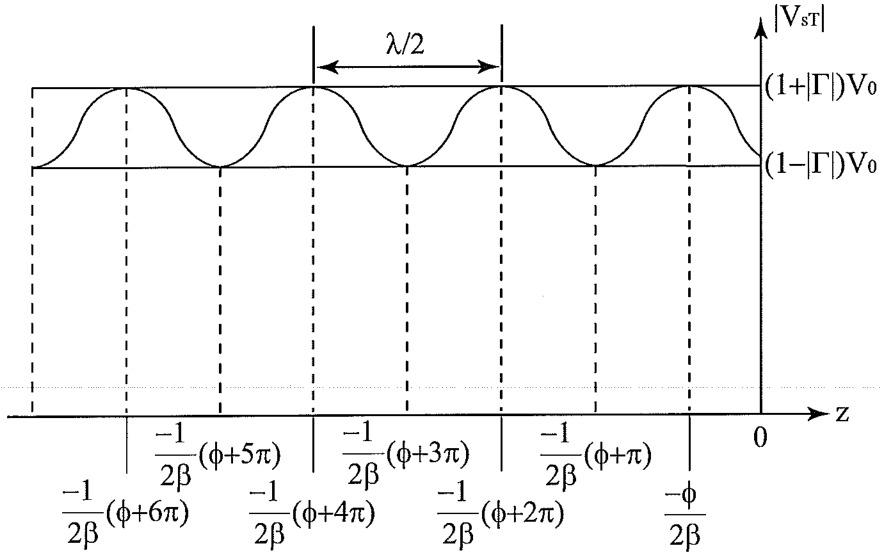


Figure 7. Standing wave along a TL. Note that ϕ in this figure is the phase of the reflection coefficient $\Gamma = |\Gamma|e^{j\phi}$.

and the position of the voltage minimum is denoted as z_{min}. Thus it can be shown that

$$z_{min} = -\frac{1}{2\beta}[\phi + (2m+1)\pi] \tag{1.75}$$

and

$$z_{max} = -\frac{1}{2\beta}[\phi + 2m\pi] \tag{1.76}$$

where $m = 0, 1, 2....$ Next, we define the VSWR (denoted as s) in the following manner:

$$s = \frac{V_{sT}(z_{max})}{V_{sT}(z_{min})} = \frac{1+|\Gamma|}{1-|\Gamma|}. \tag{1.77}$$

A few examples of a standing wave are illustrated in Fig. 8. To understand the maximum and minimum values of the voltage along the TL we start with the following expression:

$$V_{sT}(z) = V_0\left(e^{-j\beta z} + |\Gamma|e^{j(\beta z+\phi)}\right). \tag{1.78}$$

We have a voltage minimum when $\beta z = 0$ and $\phi = \pi$. This then reduces (6.28) to $V_{sT}(z_{min}) = V_0(1 - |\Gamma|)$. Similarly, we have a voltage maximum when $\beta z = 0$ and $\phi = 0$. This then reduces (6.28) to $V_{sT}(z_{max}) = V_0(1 + |\Gamma|)$. Then, if the TL is matched (i.e., $Z_0 = Z_L$) then $\Gamma = 0$ and $V_0(1 + |\Gamma|) = V_0(1 - |\Gamma|) = V_0$. This results in the constant voltage along the TL shown in Fig. 8 a). If $Z_L = 0$ (i.e., short circuit), then $|\Gamma| = -1$. This then gives a maximum and minimum voltage of $(1 + 1)V_0 = 2V_0$ and $(1 - 1)V_0 = 0$, respectively. This is illustrated in Fig. 8 b). Finally, if $Z_L = \infty$ (i.e., open circuit) then $|\Gamma| = 1$. This then gives a maximum and minimum voltage of $(1 + 1)V_0 = 2V_0$ and $(1 - 1)V_0 = 0$, respectively. This is illustrated in Fig. 8 c).

We can also see that the standing waves in Fig. 8 b) and c) have a period of $\lambda/2$. For example, to calculate the position of the first minimums and maximums we use the expressions in (6.19) and (6.20), respectively. First, if $\phi = \pi$, then (6.19) and (6.20) reduce to $-\lambda/2$ and $-\lambda/4$, respectively. These computations are illustrated in Fig. 8 b). Second, if $\phi = 0$, then (6.19) and (6.20) reduce to $-\lambda/4$ and 0, respectively. These computations are illustrated in Fig. 8 c).

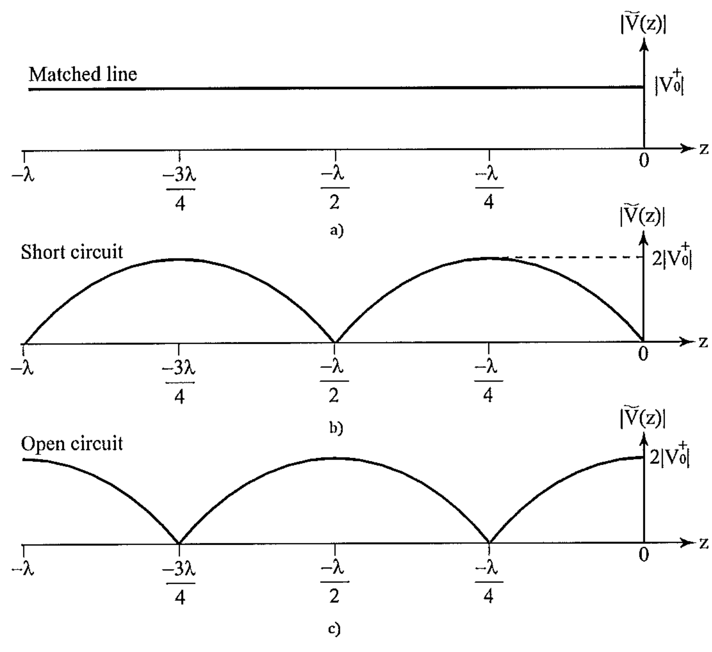


Figure 8. a) Standing wave along the TL for $Z_L = Z_0$; b) standing wave along the TL for $Z_L = 0$; c) standing wave along the TL for $Z_L = \infty$.

1.9. Examples: Standing Wave Examples

1.9.1. Example 1

A 50 Ω transmission line is terminated with a load of $Z_L = 100 + j50\ \Omega$. Find the voltage reflection coefficient and the voltage standing wave ration (VSWR).

Solution: From (1.47) we have

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 50}{100 + j50 + 50} = .45\angle 26.6.$$

Next, using (6.27) we get

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.45}{1 - 0.45} = 2.6.$$

1.9.2. Example 2

A 140 Ω lossless transmission line is terminated with a load impedance of $Z_L = 280 + j182\ \Omega$. If $\lambda = 72$ cm, find (a) Γ, (b) s and (c) the first z_{min} and z_{max}.

Solution: From (1.47) we have

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{280 + j182 - 140}{280 + j182 + 140} = .5\angle 29.$$

Next, using (6.27) we get

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.5}{1 - 0.5} = 3.0.$$

Finally, $\beta = 2\pi/\lambda = 8.72$ m. Using (6.19) and (6.20) gives $z_{min} = \frac{1}{2*8.72}(29 * \frac{\pi}{180} + \pi) = 20.9$ cm and $z_{max} = \frac{1}{2*8.72}(29 * \frac{\pi}{180}) = 2.9$ cm.

1.10. Transmission Line of Finite Length

Now we need to analyze the TL while including the numerous forward and backward reflected waves. To do this consider the TL in Fig. 9. From before, we have $V_{sT}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$ which represents the total voltage along the TL. We also have $I_{sT}(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}$ which represents the total current along the TL. Using these two expressions, we define the *wave impedance* as

$$\begin{aligned} Z_W(z) &= \frac{V_{sT}(z)}{I_{sT}(z)} = \frac{V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}{I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z}} \\ &= Z_0 \left[\frac{Z_L \cos\beta z - jZ_0 \sin\beta z}{Z_0 \cos\beta z - jZ_L \sin\beta z} \right]. \end{aligned} \tag{1.79}$$

Evaluating (6.29) at $z = -l$ gives

$$Z_{in} = Z_W(-l) = Z_0 \left[\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l} \right]. \tag{1.80}$$

Next, if $\beta l = \frac{2\pi}{\lambda}\frac{m\lambda}{2} = m\pi$ where $m = 0, 1, ...$ then $Z_{in}(l = m\lambda/2) = Z_L$. Also, if $\beta l = \frac{2\pi}{\lambda}(2m+1)\frac{\lambda}{4} = (2m+1)\pi/2$ where $m = 0, 1, ...$ then $Z_{in}(l = \lambda/4) = Z_0^2/Z_L$. The last expression is used for the design of quarter wave transformers.

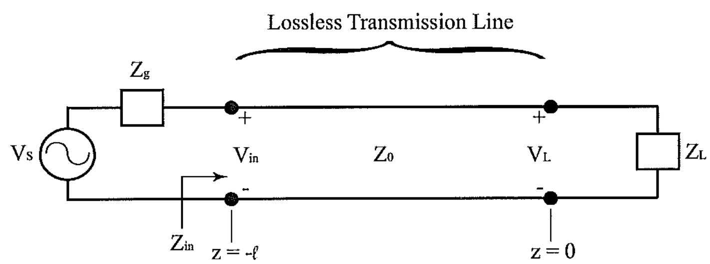


Figure 9. General lossless TL in steady state.

1.11. Examples: Finite Length Transmission Lines

1.11.1. Example 1

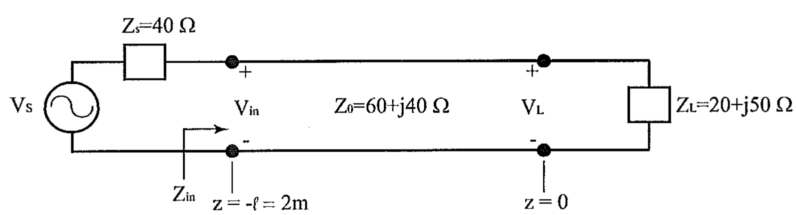


Figure 10. TL for example 1.

A TL operating at $\omega = 10^6$ rad/s has the following constants: $\alpha = 8$ dB/m, $\beta = 1$ rad/m,

Z_0=60+j40 Ω and is 2m long. If $V_s = 10\angle 0$, $Z_s = 40\ \Omega$ and $Z_L = 20 + j50\ \Omega$ find:

a) Z_{in}

b) I_{in}

Solution: Neglecting α gives

a)

$$\begin{aligned} Z_{in} = Z_W(-l) &= Z_0 \left[\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}\right] \\ &= (60 + j40)\left[\frac{(20 + j50)\cos(2) + j(60 + j40)\sin(2)}{(60 + j40)\cos(2) + j(20 + j50)\sin(2)}\right] \\ &= 57.28 - j2.11 \end{aligned}$$

b)

$$\begin{aligned} I(-l) &= \frac{V_g}{Z_{in} + Z_g} \\ &= \frac{10\angle 0}{40 + j57.28 - j2.11} \\ &= 102.7\angle 1.24mA. \end{aligned}$$

1.11.2. Example 2

Now consider the TL with $Z_0 = 300\Omega$. The load is two 300 Ω resistors and one capacitor with $Z_c = -j300\Omega$ all connected in parallel. Calculate Z_{in}, s, Γ and P_L for $l = 2m$, $\nu_p = 2.5 \times 10^8 m/s$, $f = 100MHz$, $Z_g = 300\Omega$ and V_s=60V.

Solution: $Z_L = 300||300||(-j300) = 150||(-j300) = 120 - j60\Omega.$ $\Rightarrow$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{120 - j60 - 300}{120 - j60 + 300} = .447\angle - 153.4,$$

and

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.616.$$

Since the TL is lossless, $\lambda = \nu_p/f = 2.5 \times 10^8/100MHz = 2.5m.$ $\Rightarrow$ $\beta = 2\pi/\lambda = 2.51rad/m.$ $\Rightarrow$

$\beta l = 5.02 rad = 287.6 deg$. Solving for Z_{in} gives

$$\begin{aligned} Z_{in} &= Z_0 \left[\frac{Z_L \cos 287.6 + jZ_0 \sin 287.6}{Z_0 \cos 287.6 + jZ_L \sin 287.6} \right] \\ &= 760.1 - j127.6\Omega. \end{aligned}$$

$\Rightarrow$

$$\begin{aligned} I_{in} &= \frac{V_s}{Z_g + Z_{in}} \\ &= \frac{60}{300 + Z_{in}} \\ &= 56.1\angle 6.86 mA. \end{aligned}$$

Since the TL is lossless,

$$\begin{aligned} P_{in} &= \frac{1}{2} I^2 R \\ &= \frac{1}{2}(56.1mA)^2 760 \\ &= 1.199W. \end{aligned}$$

$\Rightarrow P_L \approx 1.2W$.

1.12. The Smith Chart

1.12.1. Introduction

To introduce the Smith chart we start with the reflection coefficient $\Gamma = (Z_L + Z_0)/(Z_L + Z_0)$. All impedance values on the Smith chart are normalized and have the following notation:

$$z_L = r + jx = \frac{Z_L}{Z_0} = \frac{R_L + jX_L}{Z_0}. \tag{1.81}$$

$\Rightarrow$

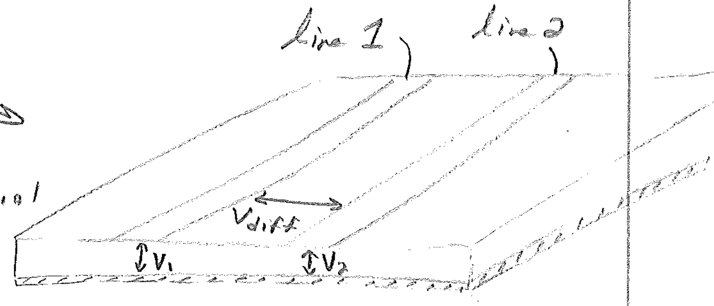
$$\Gamma = \frac{z_L - 1}{z_L + 1} \tag{1.82}$$

Differential Pairs & Differential Impedance

Differential signaling has the following advantages –

- less gnd. bounce, rail collapse + EMI.
- higher gain differential amps.
- robust propagation over tightly coupled lines.
- cost effective twisted pair can be used over long distances.

- Differential signaling is achieved w/ two single ended voltages applied to two different conductors.



One example of a differential signal is low-voltage differential signals (LVDS). ⇒ the differental voltage is $\boxed{V_{diff} = V_1 - V_2}$. We also have the common signal. This is the average voltage on each line: $\boxed{V_{comm} = \frac{1}{2}(V_1 + V_2)}$.

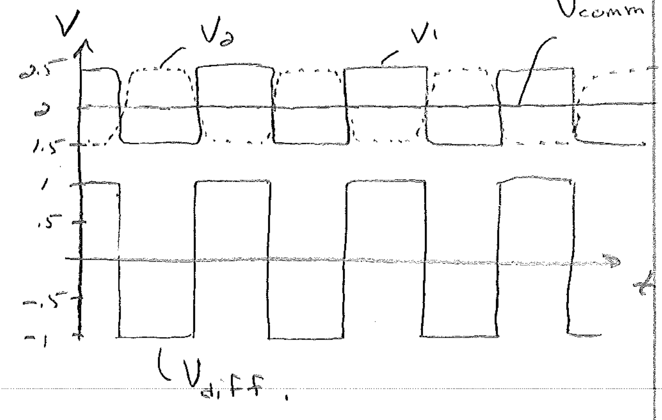
carries information

$$\Rightarrow \quad V_1 = V_{comm} + \frac{1}{2} V_{diff} \quad +$$

$$V_2 = V_{comm} - \frac{1}{2} V_{diff}.$$

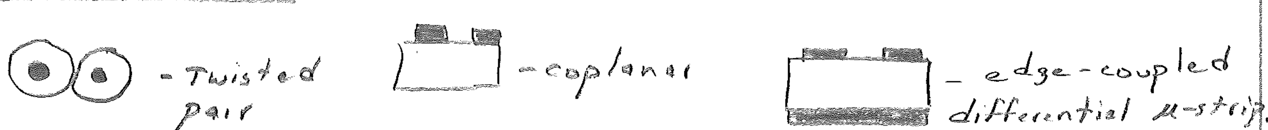
Under ideal conditions the common mode voltage is constant.

However, if the voltage path is not ideal, there may exist unwanted changes in V_1 or V_2. ∴



If V_{comm} gets too high, it may saturate the input of an amplifier or cause radiation.

<u>Differential Pair</u>



Here are 5 - important rules for designing a high BW diff. pair:

1) Design w/ a uniform conductor cross-section (minimize reflections)
2) Match the two time delays on the line.
3) The lines should by symmetric.
4) Each line should have the same length.
5) Maximize the coupling for robust lines.

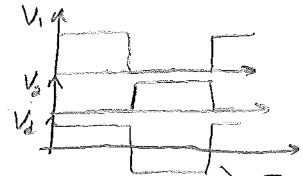
Differential Impedance

- This is very important for diff. signals.

- Differential impedance w/ no coupling. -

Assum the spacing between two diff. lines is large enough ∋ the coupling is minimized.

$\Rightarrow I_1 = \frac{V_1}{Z_0}$ where I_1 is the current into one signal line + out its return. If we assume identical lines then: ↗ For

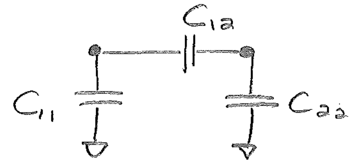


$$Z_{diff} = \frac{V_{diff}}{I_1} = \frac{2V_1}{I_1} = \boxed{2Z_0}$$

Twice the swing ⇒ $2Z_0$ to maintain $I_1 + I_2$.

- Impact of coupling -

As mentioned above, it is desirable to couple the differential TLs. However, this does effect Z_0 of each individual TL b/c the mutual cap. and ind. increases.



- Next, if two lines are driven w/ a diff. signal, $V_{12} = 2V_{11}$ where V_{12} is the diff voltage $= (V_{11} - V_{22})$ between the lines + V_{11} (V_{22}) is the line 1 (2) voltage.

$$\Rightarrow I_1 = \underbrace{v}_{\text{signal speed}} \cdot \underbrace{RT}_{\text{rise time}} \cdot \left(C_{11} \frac{dV_{11}}{dt} + C_{12} \frac{dV_{12}}{dt} \right) \approx C_{11} V_1 + 2C_{12} V_1$$

$\Rightarrow I_1 = V_1 (C_{11} + 2C_{12})$ shows that line one will have a higher current to drive the higher cap. of the single ended line when the lines are driven w/ opposite signals.

- Next, suppose the lines are driven w/ the exact same signals. $\Rightarrow V_{12} = 0$.

$\Rightarrow I_1 = V \cdot RT \cdot C_{11} \frac{dV_{11}}{dt} \sim C_{11} V_1$.

Comparing $I_{1,diff} = V_1 (C_{11} + 2C_{12})$ +

$I_{1,same} = V_1 C_{11}$

we can see that $I_{1,diff} \neq I_{1,same}$ $\Rightarrow$ The char. imp. of a single line changes w/ different signaling techniques!

- In summary - when the traces are closer than $\approx$ 3-line widths, the adjacent traces will affect the char. Imp. of the first trace.

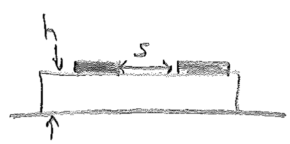
Show. Fig. 11-13 on p. 492 + Fig 11-14 on p. 493.

- At this point, ∃ only one reasonably good approx. of calculating $Z_{diff.}$ of either an edge-coupled μ-strip or an edge-coupled stripline. These results are based on curve fitting on measured data.

For edge-coupled μ-strip on FR4,

$$Z_{diff} = 2 \cdot Z_0 \left[1 - 0.48 \exp\left(-0.96 \, s/h\right)\right]$$

where - Z_0 is the char. imp. of an uncoupled single line.



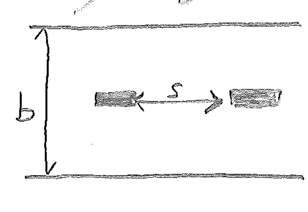
- s is the edge to edge spacing in mils.
- h is the diel. thickness in mils.

For an edge-coupled stripline on FR4

$$Z_{diff} = 2 \cdot Z_0 \left[1 - 0.37 \exp(-2.9 \, s/b)\right]$$

where - s is the edge-to-edge spacing in mils

- b is the diel. thickness.



- Show. Fig. 11-15 on p. 495.

S-parameters

- The s-parameters have become the universal standard to describe interconnects.

- In the SI world, s-parameters are called behavioral models.

- A behavioral model describes how an interconnect interacts w/ an incident waveform.

- Fundamentally, the s-parameters describe how precision waveforms, like sine waves, scatter from the ends of the interconnect.

- The term S-parameters is short for scattering parameters.

- Each s-parameter is the ratio of the output sine wave to the input:

$$S = \frac{\text{output sine wave}}{\text{input sine wave}} \in \mathbb{C}.$$

$\Rightarrow S_{dB} = 20\log(|S|)$ where S_{dB} is the dB value of $|S|$.

- Note, for the Author it is assumed that $\underline{Z_0 = 50\Omega}$.

- consider: Port 1 •————• Port 2

  Port 3 •————• Port 4

$\downarrow$

$$\Rightarrow \quad S_{kj} = \frac{\text{Sine wave out port } k}{\text{Sine wave in port } j}.$$

S_{kj}: k = output port, j = Input port.

- Insertion loss is a measure of what is lost from the signal when the interconnect is "inserted" between the two ports of a network analyzer.

- The return loss is what is returned to the incident port & lost to the transmitted signal.

- Show Fig. 12-16 on p. 578 & Fig. 12-17 on p. 579.

- In the time domain, S_{11} is viewed as a step response. The S_{21} is viewed as a step edge & has info. on how the leading or falling edge is distorted by the signal.

- Eye diagrams

- First, a pseudo random bit stream is sytlesieed usig an ideal square wave, w/ some RT.
- This wave form is then convolved w/ the impulse response of the interconnect to get an output.
- The result is how the interconnect would treat a waveform + displayed as an eye diagram.

-Show Fig. 12-40 + 12-41 on p. 612 + 613, resp.

Power Distribution Network (PDN)

-Again, the PDN consists of all those interconnects from the voltage regulators to pads on the chip.

- The primary job of the PDN is to keep a constant supply voltage on the pads of the chips within a certain tolerance.

- Secondly, the same PDN may provide a low impedance return for signals.

- Since the PDN has an impedance,

$$V(f) = I(f)\,Z(f)$$

where $V(f)$ is the voltage amp. in the freq. domain

$I(f)$ is the current spectrum drawn by ~~the chip.~~

$Z(f)$ is the impedance profile of the PDN as seen by the chip loads.

$\therefore$ to minimize the drop from the PDN, Z_{PDN} must be minimized. For a certain ripple, a <u>target impedance</u> is defined.

$$\Rightarrow \quad V_{ripple} > V_{PDN} = I(f)\,Z_{PDN}(f).$$

$$\Rightarrow \quad Z_{target}(f) = Z_{PDN}(f) < \frac{V_{ripple}}{I(f)}.$$

where - V_{ripple} is the voltage noise tolerance for the chip in Volts.

- V_{PDN} is the voltage noise drop across the PDN interconnects in Volts.

- $I(f)$ is the current spectrum drawn by the chip in Amps.

- Z_{PDN} is the impedance profile of the

PDN as seen by the chip pads in ohms.

Z_{target} is the maximum allowable impedence of the PDN in ohms.

∴ the fundamental guiding principle behind the designs of PDN is to keep the impedance of the chip as low as possible.

⇒ The three most important guidlines in designing PDNs are:

1) Use pwr. + gnd. planes on adjacent layers, with as thin a dielectric as possible + bring them as close to the surface of the board stack-up as possible (closer to the load).

2) Use as short + wide as possible surface trace between the decoupling cap. pads + the vias to the buried pwr. + gnd. plane cavity.

3) Use Spice to determine the optimum number of capacitors + their values to bring the impedance profile below the target value.

- When ever possible, the peak transient current should be used to estimate the target impedance, otherwise max power can be used.

- A good estimate for transient current is:

$$I_{transient} \sim \tfrac{1}{2} I_{max}.$$

If max pwr. is used,

$$I_{max} = \frac{P_{max}}{V_{dd}} \leftarrow \text{rail voltage in Volts.}$$

$$\Rightarrow Z_{target}(f) < \frac{V_{dd} \cdot \text{ripple }\%}{I_{transient}} = \frac{2\, V_{dd} \cdot \text{ripple }\% \cdot V_{dd}}{P_{max}}.$$

For example, if the ripple is 5%, the target impedance is $\boxed{Z_{target}(f) = 0.1 \times \frac{V_{dd}^2}{P_{max}}}$.

Show Fig. 13-8 on p. 629.

~~CHAPTER 6. TOPICS ON ELECTROMAGNETIC COMPATIBILITY~~

Cross Talk

6.1. Coupling Between Transmission Lines

6.1.1. General Problem

To investigate the coupling between two TLs we start with the general problem defined in Fig. 69. The problem insists of two parallel TLs above a common ground plane. The TL with the source is called the generator conductor and the TL with the two resistive loads is called the receptor conductor. R_s is the source resistance, R_L is the load resistance, R_{NE} is the near-end load on the receptor conductor and R_{FE} is the far-end load on the receptor conductor. The receptor conductor has a length L.

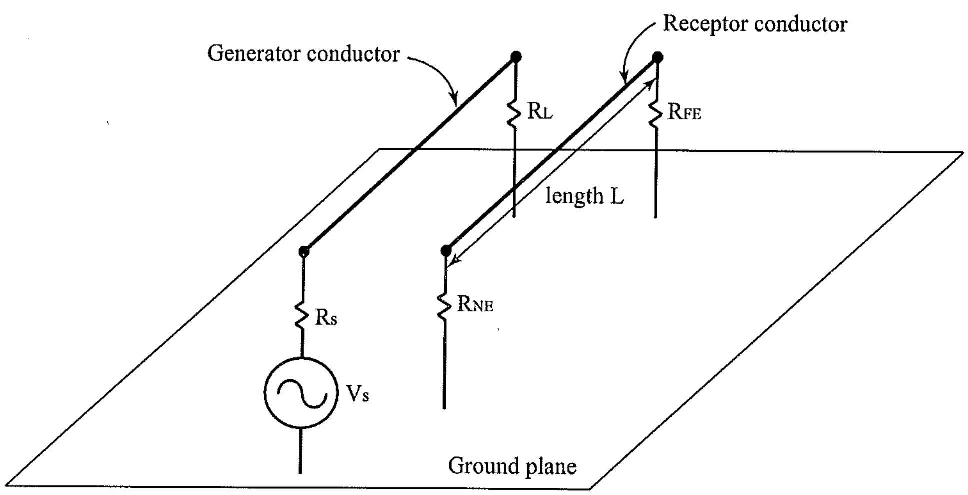


Figure 69. General TL coupling problem.

Next, we can derive an equivalent circuit of the coupled TLs. By looking at one small section of the coupled TLs we will be able to describe the interaction between the two TLs. The equivalent circuit for one section of coupled TLs is shown in Figs. 70. We want to simplify the analysis of the coupling between the two TLs in Fig. 70 to the following two cases:

1) capacitive coupling (occurs in high impedance circuits)

2) inductive coupling (occurs in low impedance circuits)

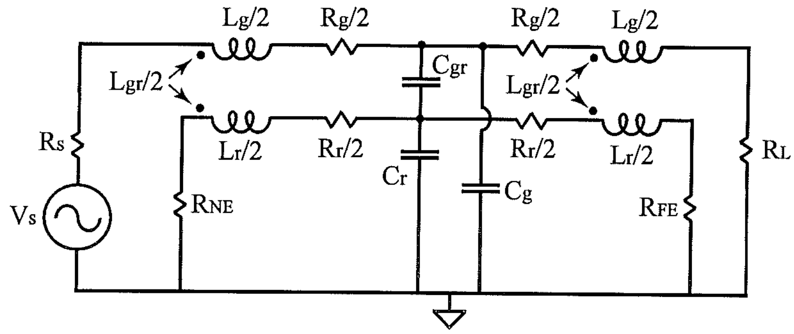
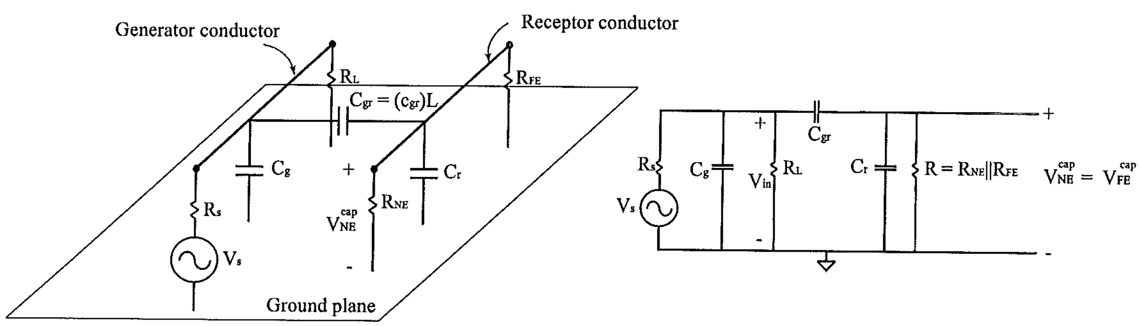


Figure 70. Equivalent circuit of the coupled transmission lines.

6.1.2. Capacitive coupling (low frequencies)

For this analysis, we assume that we have high impedance loads on the coupled TLs in Fig. 69. If this is the case, then the coupling between the TLs is mostly capacitive. This reduces the equivalent circuit in Fig. 70 to the equivalent circuits shown in Figs. 71 a) and b).



a) b)

Figure 71. a) Capacitive coupling between the coupled transmission lines and b) the equivalent circuit of the capacitively coupled transmission lines.

Next, we want to write the voltages induced on R_{NE} and R_{FE} by V_s. Evaluating the equivalent circuit in Fig. 71 b) gives

$$\begin{aligned} V_{NE}^{cap} = V_{FE}^{cap} &= V_{in}\left(\frac{R||\frac{1}{j\omega C_r}}{R||\frac{1}{j\omega C_r} + \frac{1}{j\omega C_{gr}}}\right) \\ &= V_{in}\left(\frac{j\omega\frac{C_{gr}}{C_{gr}+C_r}}{j\omega + \frac{1}{R(C_{gr}+C_r)}}\right). \end{aligned} \tag{6.1}$$

Next, for low frequencies we have the following assumption:

$$\omega << \frac{1}{R(C_{gr} + C_r)}. \tag{6.2}$$

This then simplifies (6.1) to

$$V_{NE}^{cap} = V_{FE}^{cap} \approx j\omega C_{gr} V_{in} R. \tag{6.3}$$

Note again that (6.3) is for high impedance circuits. We can also write

$$V_{in} = V_s \left(\frac{Z_{in}}{Z_{in} + R_s} \right) \tag{6.4}$$

and

$$Z_{in} = \left(\frac{1}{j\omega C_g} \right) || R_L || \left[\frac{1}{j\omega C_{gr}} + R || \left(\frac{1}{j\omega C_r} \right) \right]. \tag{6.5}$$

But, for low frequencies we can approximate a capacitor as an open. This then gives

$$V_{in} \approx V_{g(DC)} \approx V_s \left(\frac{R_L}{R_L + R_s} \right). \tag{6.6}$$

6.1.3. Inductive coupling (low frequencies)

In this section we want to look at the low frequency inductive coupling between the TLs. For this case we simply open all capacitors in Fig. 70. This then results in the equivalent circuit shown in Fig. 72. The near-end voltage can be written as $V_{NE}^{ind} = -I_r R_{NE}$ and the far-end voltage can be written as $V_{FE}^{ind} = I_r R_{FE}$. Next, KVL for the receptor gives $I_r R_{NE} + j\omega L_r I_r + R_{FE} I_r + j\omega L_{gr} I_g = 0$. $\Rightarrow$

$$I_r = \frac{-j\omega L_{gr} I_g}{(R_{NE} + R_{FE}) + j\omega L_r}. \tag{6.7}$$

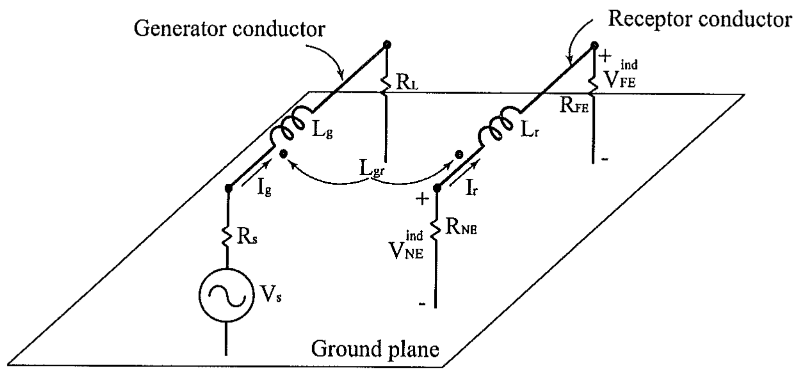


Figure 72. The equivalent circuit of the inductively coupled transmission lines.

Next, KVL around the generator gives $-V_s + R_s I_g + j\omega L_g I_g + I_g R_L + j\omega I_r L_{gr} = 0.$ $\Rightarrow$

$$\begin{aligned} I_g &= \frac{V_s}{(R_s+R_L)+j\omega L_g} + \frac{-j\omega L_{gr}}{(R_s+R_L)+j\omega L_g}\frac{-j\omega L_{gr} I_g}{(R_{NE}+R_{FE})+j\omega L_r} \\ &= \frac{V_s}{(R_s+R_L)+j\omega L_g} + \frac{-j\omega L_{gr}}{(R_s+R_L)+j\omega L_g} I_r. \end{aligned} \tag{6.8}$$

Next, if we assume that $\omega L_r << R_{NE} + R_{FE}$, $\omega L_g << (R_s + R_L)$ and a weak coupling condition of $(\omega L_{gr})^2 << (R_s + R_L)(R_{NE} + R_{FE})$ then

$$I_g \approx \frac{V_s}{R_s + R_L} = I_g(DC) \tag{6.9}$$

and

$$I_r \approx \frac{-j\omega L_{gr}}{R_{NE} + R_{FE}} I_g(DC). \tag{6.10}$$

Then

$$V_{NE}^{ind} \approx j\omega L_{gr} \frac{R_{NE}}{R_{NE} + R_{FE}} I_g(DC) \tag{6.11}$$

and

$$V_{FE}^{ind} \approx -j\omega L_{gr} \frac{R_{FE}}{R_{NE} + R_{FE}} I_g(DC). \tag{6.12}$$

6.1.4. Equations for both inductive and capacitive coupling

Noting that both V^{ind} and V^{cap} are proportional to $j\omega$ and V_s we can write the results as

$$\frac{V_{NE}}{V_s} = j\omega\left(M_{NE}^{ind} + M_{NE}^{cap}\right) \tag{6.13}$$

and

$$\frac{V_{FE}}{V_s} = j\omega\left(M_{FE}^{ind} + M_{FE}^{cap}\right) \tag{6.14}$$

where

$$M_{NE}^{ind} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{gr}}{R_s + R_L}, \tag{6.15}$$

$$M_{FE}^{ind} = \frac{-R_{FE}}{R_{NE} + R_{FE}} \frac{L_{gr}}{R_s + R_L} \tag{6.16}$$

and

$$M_{NE}^{cap} = M_{FE}^{cap} = (R_{NE}||R_{FE})\frac{R_L C_{gr}}{R_s + R_L}. \tag{6.17}$$

This then results in the inductive/capacitive model shown in Fig. 73 where

$$V_g(DC) = V_s \frac{R_L}{R_L + R_s} \tag{6.18}$$

and

$$I_g(DC) = \frac{V_s}{R_s + R_L}. \tag{6.19}$$

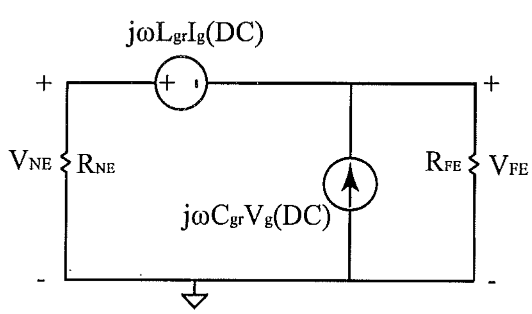


Figure 73. The equivalent inductive/capacitive model of the coupled transmission lines.

Note that the equivalent circuit model shown in Fig. 73 breaks down for TL lengths longer than $\approx 0.2\lambda$. Therefore, if the frequency is high enough then a smaller segment of the TL should be analyzed or a more accurate distributive model of the coupled TL should be used.

Next, we consider the problem in Fig. 69 for the case when the ground plane has finite conductivity. We represent the loss with a lumped resistor $R_o = r_o l$ where l is the length of the TL and r_o is the per-unit loss of the ground plane. The problem with R_o defined is shown in Fig. 74. R_o is called the common impedance and V_o is called the common impedance voltage where $V_o = r_o I_{ref}$. $\Rightarrow I_{ref} = I_r + I_g \approx I_g(DC)$ at low frequencies. $\Rightarrow$

$$V_o \approx V_s\left(\frac{R_o}{R_s + R_L}\right). \tag{6.20}$$

This voltage appears on the receptor circuit also. Therefore, this contributes to the near- and far-end noise voltages at low frequencies. This then gives:

$$\frac{V_{NE}^{ci}}{V_s} = M_{NE}^{ci} \tag{6.21}$$

and

$$\frac{V_{FE}^{ci}}{V_s} = M_{FE}^{ci} \tag{6.22}$$

where

$$M_{NE}^{ci} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{R_o}{R_s + R_L} \tag{6.23}$$

and

$$M_{FE}^{ci} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{R_o}{R_s + R_L}. \tag{6.24}$$

With this notation, we have the following final coupling equations that include the common impedance noise voltages:

$$\frac{V_{NE}}{V_s} = j\omega\left(M_{NE}^{ind} + M_{NE}^{cap}\right) + M_{NE}^{ci} \tag{6.25}$$

and

$$\frac{V_{FE}}{V_s} = j\omega\left(M_{FE}^{ind} + M_{FE}^{cap}\right) + M_{FE}^{ci} \tag{6.26}$$

This results in the equivalent circuit show in Fig. 75.

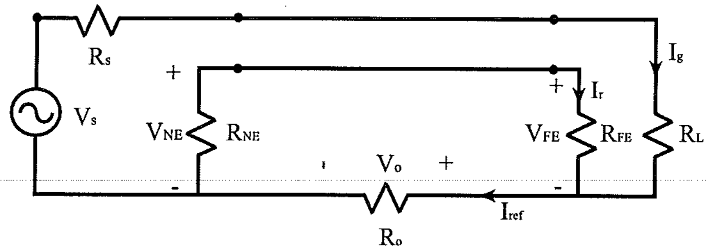


Figure 74. Coupled transmission lines with a lossy reference conductor.

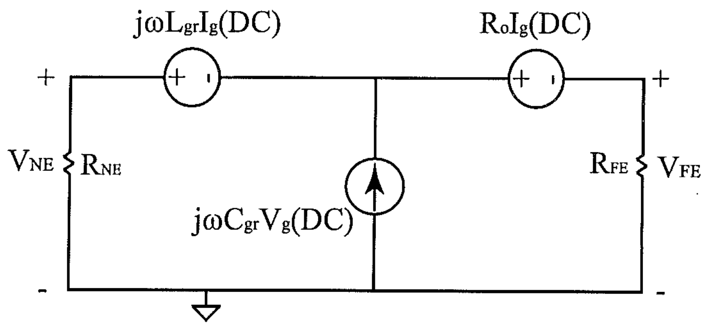


Figure 75. The equivalent inductive/capacitive model of the coupled transmission lines with a lossy reference conductor.

6.2. Shielding

6.2.1. Reducing capacitive coupling

In this section we investigate the use of shielding to reduce the coupling between the TLs in Fig. 69. From (6.3) we can see that we can reduce the capacitive coupling by :

1) reducing frequency

2) reducing R_{NE} and/or R_{FE}

3) reducing $V_g(DC)$ by reducing V_s

4) reducing C_{gr} by increasing the spacing between the traces and shielding

For this section we will focus on using a shield around the receptor to reduce the capacitive coupling. This problem is defined in Fig. 76 where V_s is the generator (source) open circuit voltage, $V_{s,o}$ is the output voltage of the source, C_g is the capacitance from the generator to the reference conductor, C_{gs} is the capacitance from the generator to the shield, C_{gr} is the capacitance from the generator to the reference conductor, C_s is the capacitance from the shield to the reference conductor, C_{rs} is the capacitance from the receptor conductor to the shield, C_r is the capacitance from the receptor to the reference conductor and V_{sh} is the voltage between the shield and reference conductor. If we ground the shield on the receptor conductor (at both ends), then we have the equivalent circuit shown in Fig. 77.

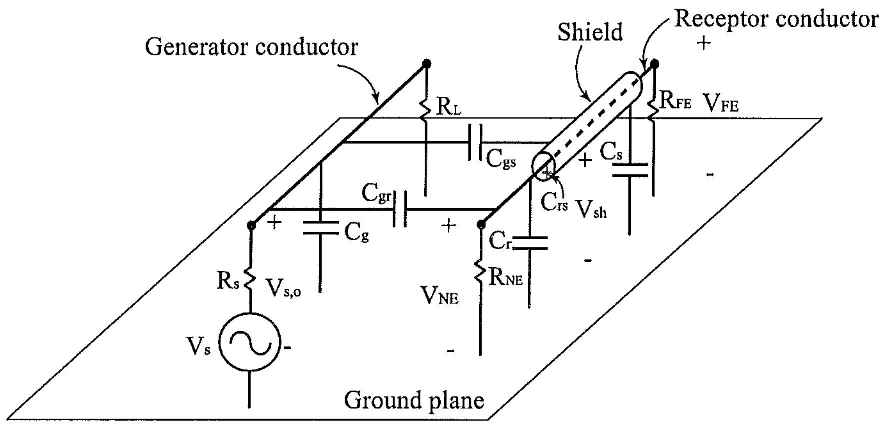


Figure 76. Capacitively coupled transmission lines with a shielded receptor.

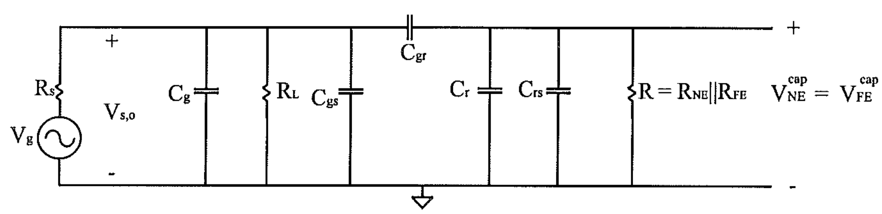


Figure 77. The equivalent circuit of the coupled transmission lines with a shielded receptor.

Using circuit analysis, the following expression can be evaluated from the circuit in Fig. 77:

$$V_{NE}^{cap} = V_{s,o}\left(\frac{R||\left(\frac{1}{j\omega(C_{rs}+C_r)}\right)}{R||\left(\frac{1}{j\omega(C_{rs}+C_r)}\right)+\frac{1}{j\omega C_{gr}}}\right) \tag{6.27}$$

or

$$V_{NE}^{cap} = V_{s,o}\left(\frac{j\omega\left(\frac{C_{gr}}{C_{gr}+C_{rs}+C_r}\right)}{j\omega + \frac{1}{R(C_{gr}+C_{rs}+C_r)}}\right). \tag{6.28}$$

At low frequencies we have $V_{s,o} \approx V_s R_L/(R_L + R_s) = V_g(DC)$ and $\omega << 1/(R(C_{gr} + C_r + C_{rs}))$. This then simplifies (6.28) down to

$$V_{NE}^{cap} = V_{FE}^{cap} \approx j\omega R C_{gr} V_g(DC). \tag{6.29}$$

Equation (6.29) is similar to (6.3), except in (6.29) C_{gr} is much less than C_{gr} in (6.3).

6.2.2. Reducing inductive coupling

When looking at shielding for inductive coupling, we need to first consider the expressions in (6.11) and (6.12). We can see that we can reduce the inductive coupling by :

1) reducing frequency

2) reducing $I_g(DC)$

3) reducing L_{gr} by

i) reducing the area between the trace and ground plane

ii) orientation

iii) a choke

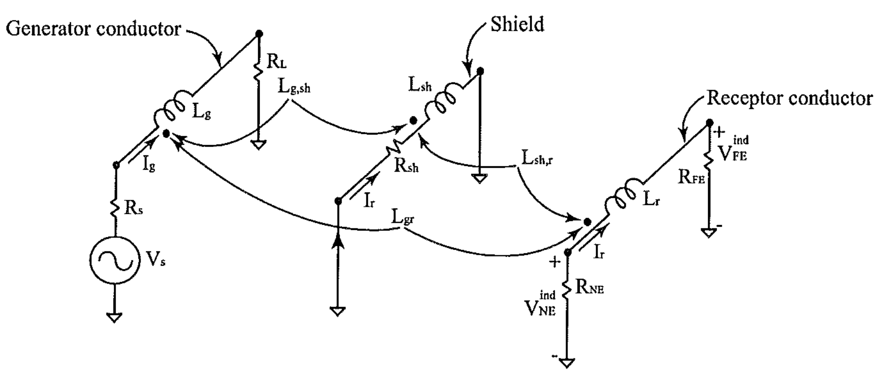


Figure 78. Inductively coupled transmission lines with a shielded receptor.

To understand the best method of shielding for inductive coupling we need to consider the problem in Fig. 78. We want to write the voltages across the near- and far-end resistors in terms of the inductive coupling and the source voltage. First, KVL around the shield loop gives: $I_{sh}R_{sh} + (j\omega L_{g,sh})I_g + (j\omega L_{sh})I_{sh} + (j\omega L_{sh,r})I_r = 0$. Solving for I_{sh} gives

$$I_{sh} = \frac{-j\omega(L_{g,sh}I_g + L_{sh,r}I_r)}{R_{sh} + j\omega L_{sh}}. \tag{6.30}$$

Next, KVL around the receptor loop gives: $I_rR_{NE}+(j\omega L_{gr})I_g+(j\omega L_{sh,r})I_{sh}+(j\omega L_r)I_r+I_rR_{FE} = 0$ where $V_{NE}^{ind} = -I_rR_{NE}$ and $V_{FE}^{ind} = -I_rR_{FE}$. Solving for I_r gives:

$$I_r = \frac{-j\omega(L_{gr}I_g + L_{sh,r}I_{sh})}{R_{NE} + R_{FE} + j\omega L_r}. \tag{6.31}$$

Next, substituting (6.30) into (6.31) gives

$$I_r = \frac{-j\omega}{R_{NE} + R_{FE} + j\omega L_r}\left[\frac{L_{gr}I_g(R_{sh} + j\omega L_{sh}) - j\omega L_{sh,r}(L_{g,sh}I_g + L_{sh,r}I_r)}{R_{sh} + j\omega L_{sh}}\right]. \tag{6.32}$$

Rearranging then gives:

$$I_r = \frac{-j\omega}{R_{NE} + R_{FE} + j\omega L_r}\left[\frac{I_g(L_{gr}R_{sh} + j\omega L_{sh}L_{gr} - j\omega L_{sh,r}L_{g,sh}) - j\omega(L_{sh,r})^2I_r}{R_{sh} + j\omega L_{sh}}\right]. \tag{6.33}$$

Now assume we have a uniform current distribution along the conductors in Fig. ??. This then gives $L_{gr} \approx L_{g,sh}$ and $L_{sh} \approx L_{sh,r}$. Rearranging (6.33) and substituting in the above assumptions gives:

$$I_r\left[1 + \frac{(\omega L_{sh,r})^2}{[R_{NE} + R_{FE} + j\omega L_r][R_{sh} + j\omega L_{sh}]}\right] = I_g\left[\frac{-j\omega L_{gr}R_{sh}}{[R_{NE} + R_{FE} + j\omega L_r][R_{sh} + j\omega L_{sh}]}\right]. \tag{6.34}$$

Solving for I_r gives

$$I_r = \frac{-j\omega I_gL_{gr}R_{sh}}{[R_{NE} + R_{FE} + j\omega L_r][R_{sh} + j\omega L_{sh}] + (\omega L_{sh,r})^2}. \tag{6.35}$$

Next, if we denote the denominator of (6.35) at $DEN = [R_{NE}+R_{FE}+j\omega L_r][R_{sh}+j\omega L_{sh}]+(\omega L_{sh,r})^2$

and assume that $L_r \approx L_{sh} \approx L_{sh,r}$ then DEN simplifies to $DEN = (R_{NE}+R_{FE})R_{sh}+j\omega L_{sh}[R_{NE}+R_{FE}+R_{sh}]$. Next, if we assume that $R_{NE}+R_{FE} >> R_{sh}$ then (6.35) simplifies to

$$I_r \approx \frac{-j\omega I_g L_{gr} R_{sh}}{(R_{NE}+R_{FE})(R_{sh}+j\omega L_{sh})}. \tag{6.36}$$

Using $I_g \approx I_g(DC) = \frac{V_s}{R_s+R_L}$. This then gives the following expressions for the near- and far-end voltages in Fig. ??:

$$V_{NE}^{ind} = -R_{NE}I_r \approx \left[\frac{R_{NE}}{R_{NE}+R_{FE}}(j\omega L_{gr}I_g(DC))\right]\left[\frac{R_{sh}}{R_{sh}+j\omega L_{sh}}\right] \tag{6.37}$$

and

$$V_{FE}^{ind} = R_{FE}I_r \approx \left[\frac{-R_{FE}}{R_{NE}+R_{FE}}(j\omega L_{gr}I_g(DC))\right]\left[\frac{R_{sh}}{R_{sh}+j\omega L_{sh}}\right]. \tag{6.38}$$

The

$$\left[\frac{R_{sh}}{R_{sh}+j\omega L_{sh}}\right] \tag{6.39}$$

term in (6.38) is referred to as the Shield Factor (SF). This term is the same in both (6.37) and (6.38). Also note that

$$SF = \frac{R_{sh}}{R_{sh}+j\omega L_{sh}} = \frac{1}{1+j\omega\frac{L_{sh}}{R_{sh}}}. \tag{6.40}$$

This then implies:

$$SF = \begin{cases} 1 & if \quad \omega << R_{sh}/L_{sh} \\ R_{sh}/j\omega L_{sh} & if \quad \omega >> R_{sh}/L_{sh} \end{cases} \tag{6.41}$$

What we see from (6.41) is that the shield has no effect on the coupling caused by the mutual inductance at low frequencies. The voltage w.r.t. frequency caused by mutual inductance is plotted in Fig. 79. We can see that the shield for the inductive coupling starts to reduce V^{ind} at approximately R_{sh}/L_{sh}. Therefore, the shield grounded at one end does not improve the inductive crosstalk. To do this we need to ground the shield at both ends. However, shields grounded at both ends are susceptible to ground loop problems.

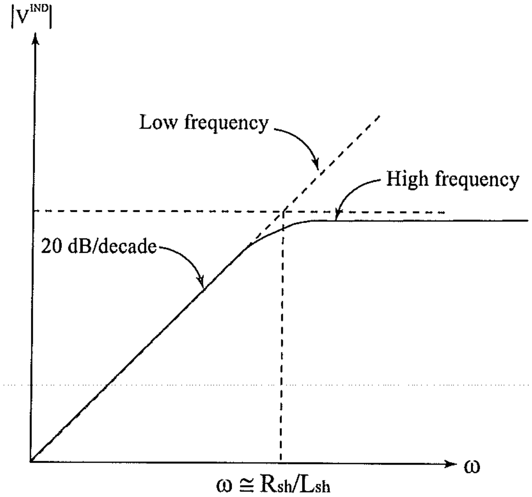


Figure 79. Induced voltage caused by inductive coupling.

6.2.3. Summary of transmission line coupling equations

Without a shield

For capacitive coupling *without a shield* we have the following equations:

$$V_{NE}^{cap} = V_{FE}^{cap} \approx j\omega C_{gr} V_{in} R. \tag{6.42}$$

For inductive coupling *without a shield* we have the following equations:

$$V_{NE}^{ind} \approx j\omega L_{gr} \frac{R_{NE}}{R_{NE} + R_{FE}} I_g(DC) \tag{6.43}$$

and

$$V_{FE}^{ind} \approx -j\omega L_{gr} \frac{R_{FE}}{R_{NE} + R_{FE}} I_g(DC) \tag{6.44}$$

where $I_g(DC) = V_s/(R_s + R_L)$.

Then to compute both the inductive and capacitive coupling between TLs *without a shield* we use the following expressions:

$$\frac{V_{NE}}{V_s} = j\omega \left(M_{NE}^{ind} + M_{NE}^{cap} \right) \tag{6.45}$$

and

$$\frac{V_{FE}}{V_s} = j\omega\left(M_{FE}^{ind} + M_{FE}^{cap}\right) \tag{6.46}$$

where

$$M_{NE}^{ind} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_{gr}}{R_s + R_L}, \tag{6.47}$$

$$M_{FE}^{ind} = \frac{-R_{FE}}{R_{NE} + R_{FE}} \frac{L_{gr}}{R_s + R_L} \tag{6.48}$$

and

$$M_{NE}^{cap} = M_{FE}^{cap} = (R_{NE}||R_{FE}) \frac{R_L C_{gr}}{R_s + R_L}. \tag{6.49}$$

Next, if we include the common impedance noise voltage *(these expressions are valid with or without a shield)* we have:

$$\frac{V_{NE}}{V_s} = j\omega\left(M_{NE}^{ind} + M_{NE}^{cap}\right) + M_{NE}^{ci} \tag{6.50}$$

and

$$\frac{V_{FE}}{V_s} = j\omega\left(M_{FE}^{ind} + M_{FE}^{cap}\right) + M_{FE}^{ci} \tag{6.51}$$

where

$$M_{NE}^{ci} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{R_o}{R_s + R_L} \tag{6.52}$$

and

$$M_{FE}^{ci} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{R_o}{R_s + R_L}. \tag{6.53}$$

With a shield

For capacitive coupling *with a shield* we have the following equations:

$$V_{NE}^{cap} = V_{FE}^{cap} \approx j\omega R C_{gr} V_g(DC). \tag{6.54}$$

For inductive coupling *with a shield* we have the following equations:

$$V_{NE}^{ind} = \left[\frac{R_{NE}}{R_{NE} + R_{FE}} j\omega L_{gr} I_g(DC)\right] SF \tag{6.55}$$

and

$$V_{FE}^{ind} = \left[\frac{-R_{FE}}{R_{NE}+R_{FE}} j\omega L_{gr} I_g(DC)\right] SF \tag{6.56}$$

where

$$SF = \frac{R_{sh}}{R_{sh}+j\omega L_{sh}} = \frac{1}{1+j\omega \frac{L_{sh}}{R_{sh}}}. \tag{6.57}$$

This then implies:

$$SF = \begin{cases} 1 & if \quad \omega << R_{sh}/L_{sh} \\ R_{sh}/j\omega L_{sh} & if \quad \omega >> R_{sh}/L_{sh} \end{cases} \tag{6.58}$$

Then to compute both the inductive and capacitive coupling between TLs *with a shield* we use the following expressions:

$$\frac{V_{NE}}{V_s} = j\omega\left(M_{NE}^{ind} + M_{NE}^{cap}\right) \tag{6.59}$$

and

$$\frac{V_{FE}}{V_s} = j\omega\left(M_{FE}^{ind} + M_{FE}^{cap}\right) \tag{6.60}$$

where

$$M_{NE}^{ind} = \frac{R_{NE}}{R_{NE}+R_{FE}} \frac{L_{gr}}{R_s+R_L} SF, \tag{6.61}$$

$$M_{FE}^{ind} = \frac{-R_{FE}}{R_{NE}+R_{FE}} \frac{L_{gr}}{R_s+R_L} SF \tag{6.62}$$

and

$$M_{NE}^{cap} = M_{FE}^{cap} = (R_{NE}||R_{FE}) \frac{R_L C_{gr}}{R_s+R_L}. \tag{6.63}$$

6.3. Twisted pair

The next TL coupling problem investigated is the twisted pair problem shown in Fig. 80. The problem consists of two conductors twisted together in the presence of a single conductor carrying a current $\widetilde{I}_G$. In this problem there exist capacitive and inductive coupling between the twisted pair and the single conductor. To represent this coupling, we define the equivalent circuit shown in Fig. 81. We will evaluate this equivalent circuit to understand how the twisted affect of the wire can reduce the inductive coupling. For this investigation we have two possible loading configurations:

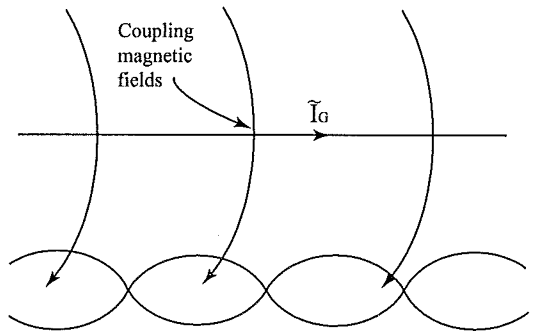


Figure 80. Twisted pair coupling problem.

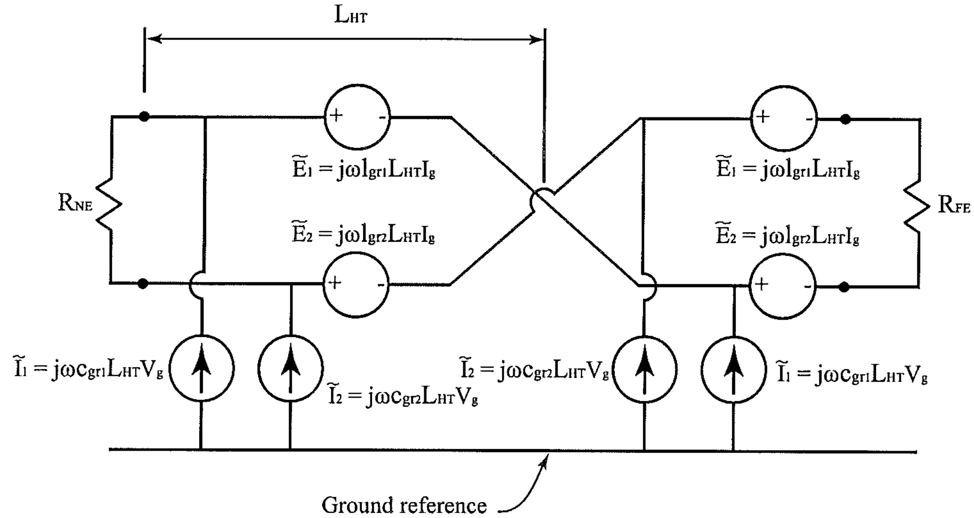


Figure 81. Equivalent circuit of the twisted pair coupling problem.

1) Balanced configuration - both wires have the same impedance to ground.

2) Unbalanced configuration - both wires do not have the same impedance to ground.

The ind./cap. model used to investigate the crosstalk in the twisted pair will depend on how the ground is connected (i.e., if a balanced or unbalanced system is used). First, if we assume an unbalanced system we get the equivalent circuits in Figs. 82 and 83. The model in Fig. 82 represents the inductive coupling and the model in Fig. 83 represents the capacitive coupling.

6.3.1. Inductive coupling - unbalanced

When evaluating the equivalent circuit in Fig. 82 using KVL we get the following equation: $\widetilde{E}_1 + \widetilde{E}_2 - \widetilde{E}_1 - \widetilde{E}_2 = 0$. This shows that for each unit of twisted wires along the twisted pair cable, we have a zero-sum value for the KVL analysis of each twist. This indicates that the inductive coupling between the single wire and the twisted pair in Fig. 80 is almost zero or negligible. This then tells us that twisted pair is useful when it is desired to reduce the crosstalk due to inductive coupling.

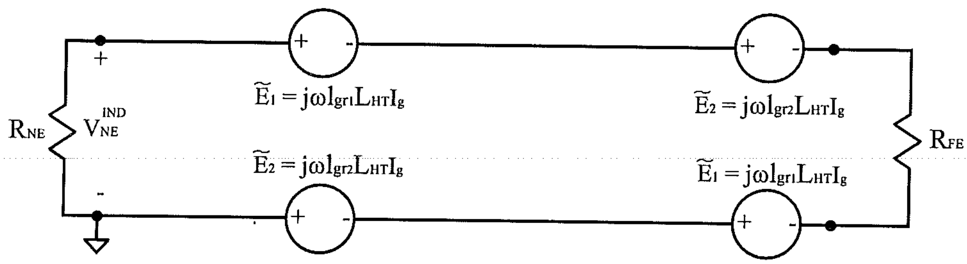


Figure 82. Equivalent circuit for the inductive coupling in the twisted pair coupling problem.

6.3.2. Capacitive coupling - unbalanced

Next, for the capacitive coupling we consider the equivalent circuit in Fig. 83. If we assume that the twisted pair is tight, then we can assume the the following: $c_{gr1} \approx c_{gr2}$. This assumption states that the capacitive between the generator in Fig. 80 and both of the conductors in the twisted pair are the same. This then reduces the circuit in Fig. 83 to the circuit in Fig. 84. We can then write the near-end and far-end voltage as

$$V_{NE}^{CAP} \approx j\omega c_{gr1} L_T N_T V_g(DC) R_{NE}||R_{FE} \tag{6.64}$$

or

$$V_{NE}^{CAP} \approx j\omega C_{gr1} V_g(DC) R_{NE}||R_{FE} \tag{6.65}$$

where L_T is the length of the full-twist, N_T is the number of twists and $C_{gr1} = c_{gr1} L_T N_T$. Therefore, using a twisted pair with unbalanced grounding reduces V^{IND} but not V^{CAP}. Finally, in general we have

$$V_{NE} \approx \frac{R_{NE}}{R_{NE} + R_{FE}} [j\omega(l_{gr1} - l_{gr2}) L_{HT}] I(DC) + j\omega C_{gr1} V_g(DC) R_{NE}||R_{FE}. \tag{6.66}$$

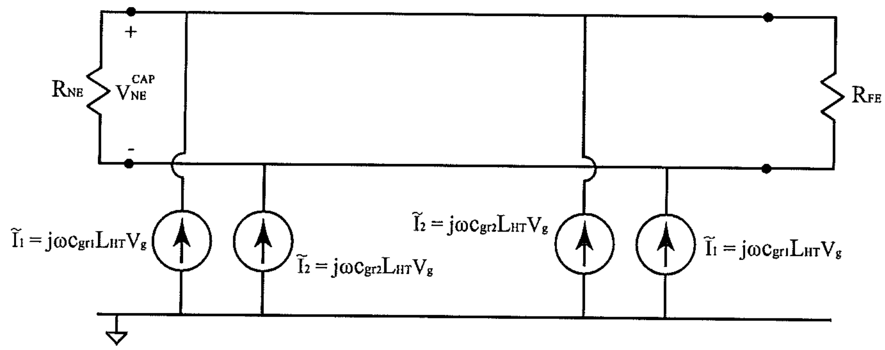


Figure 83. Equivalent circuit for the capacitive coupling in the twisted pair coupling problem.

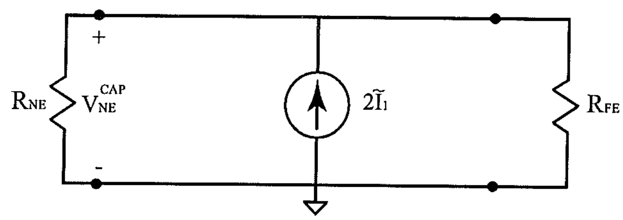


Figure 84. Reduced equivalent circuit for the capacitive coupling in the twisted pair coupling problem.

6.3.3. Balanced case

For this section we look at the balanced case (Fig. 85). The equivalent circuit for capacitive coupling for this case is shown in Fig. 86. *The inductive coupling is the same as for the unbalanced case.* We can see that the near-end voltage is zero for an even number of twists and close to zero for an odd number of twists. Therefore, for the balance case the capacitive coupling is essential zero. In summary, if possible it is best to use the twisted pair in a balanced case because both the inductive and capacitive coupling is minimized.

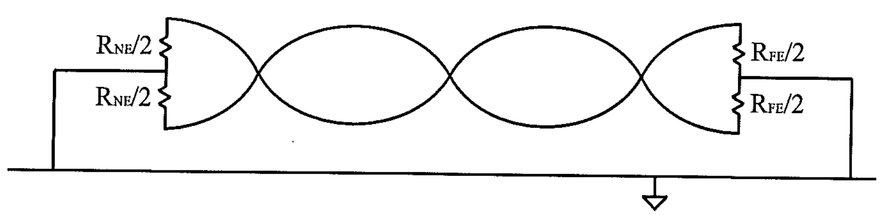


Figure 85. The twisted pair coupling problem with a balanced load.

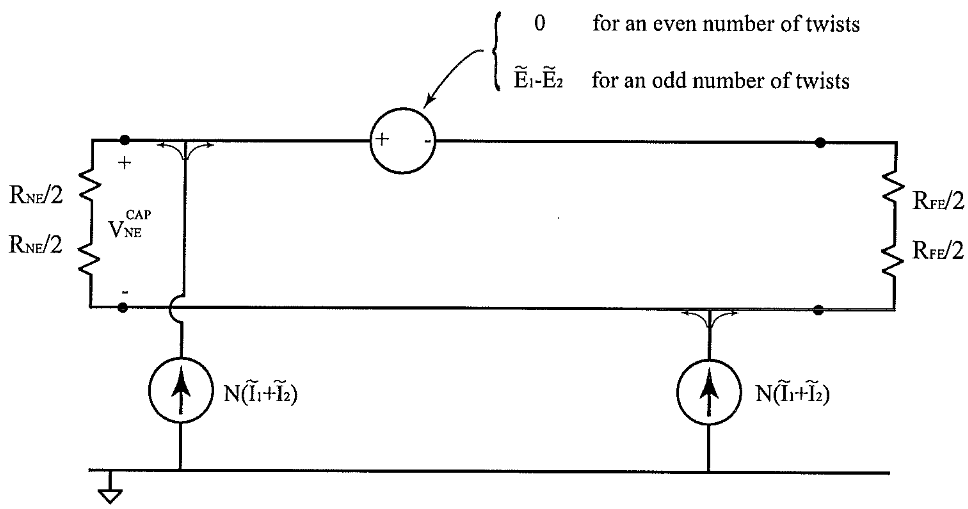
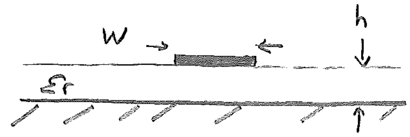


Figure 86. The equivalent circuit for the capacitive coupling in the twisted pair coupling problem with a balanced load.

Characteristic Impedance of a microstrip TL

$$Z_o = \frac{87\Omega}{\sqrt{1.41+\varepsilon_r}} \ln\left[\frac{5.98h}{.8W+t}\right]$$



where Z_o is the char. imp.

h is the diel. thickness (mils)

W is the line width (mils).

t is the metal thickness (mils).

ε_r is the diel. constant.