Circuit Analysis

- Foundation includes charges -
  - ↳ charge is bipolar - meaning we have both pos. + neg. charges
  - Unit of charge is a Coulomb.
    - ↳ There are 6.24×10^{18} charges in a single coulomb.

- There are two orientations of charges we consider: separation + movement

separation → creates electrical force between charges - known in a ckt as a voltage or electrical potential.

movement → creates electric fluid between points - known as an electric current or simply current.

Voltage - Defined as the energy per unit charge created by a separation of

$$\boxed{V = \frac{dw}{dq}} \; (V) \text{ or } \frac{(J)}{(C)}$$

where V = voltage in volts (V)
W = energy in joules (J)
q = charge in coulombs

<u>Current</u> - Defined as Coulombs per unit time mostly or

$$i = \frac{dq}{dt} \, (A) \text{ or } \frac{(C)}{(s)}$$

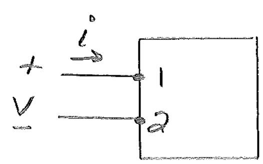
where i = current in amperes

q = charge in coulombs

t = time in seconds

<u>Passive Sign Convention</u>

When the Voltage drop is in the same direction as the current, then we use



a positive sign in any expression that relates V to i. o.w. we use a negative sign.

<u>Power and Energy</u>

Power is defined as the energy expended (or absorbed) per unit time or

$$p = \frac{dW}{dt} \Rightarrow 1W = 1\frac{J}{s}.$$

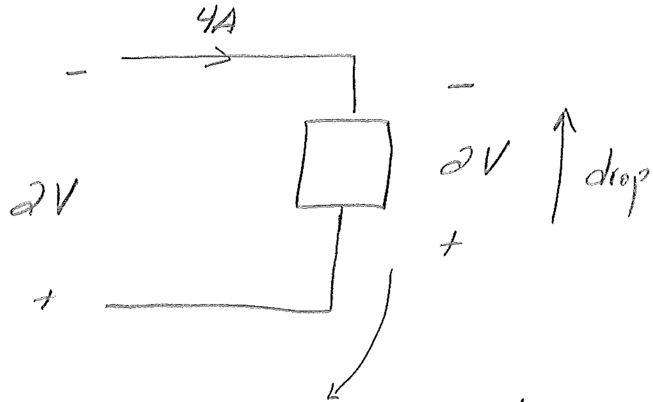
where p = pwr. in Watts (W).

$$\Rightarrow p = \frac{dW}{dt} = \frac{dW}{dq}\frac{dq}{dt}$$

$$\Leftrightarrow \boxed{P = Vi \ (W)} \leftarrow \text{Power equation.}$$

Now we use the passive sign convention for power computations

Ex 1:



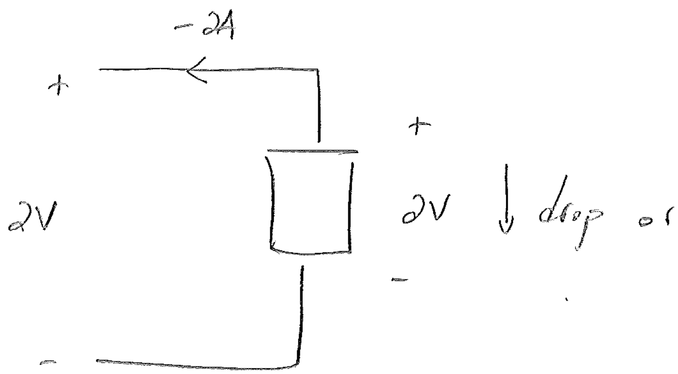
Is the elt. absorbing or supplying pwr?

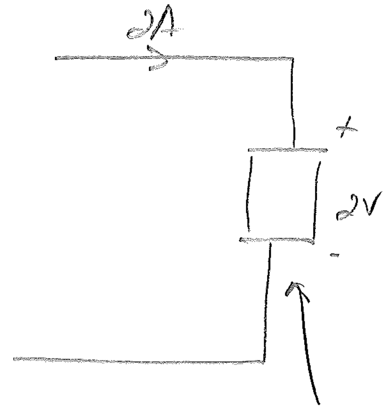
$P = Vi$ relates voltage & current. $\Rightarrow P = -Vi$

$= -8W.$

$\Rightarrow$ elt. is supplying pwr.

Ex 2:





element

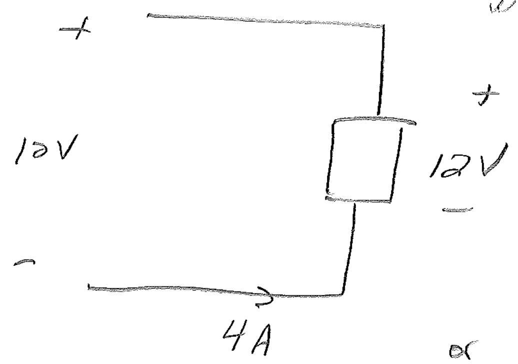
$\Rightarrow P = -Vi$

$= -(2 \cdot -2)$

$= 4W \Rightarrow$ absorbing pwr.

Since 2A is going into the pos. term. of an elt., the elt. is absorbing 4W of pwr.

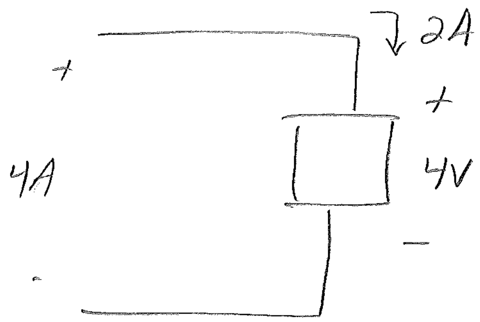
Ex 3 Pwr absorbed or supplied by elt.



⇒ Supplying $P = 4 \cdot 12$
$= 48W$ of pwr.

or absorbing $-48W$ of pwr.

Ex. 4 pwr. abs. or supplied by elt.



Pwr. absorbed is 8W or $-8W$ supplied.

Circuit Element Models

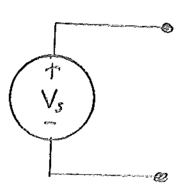
2 model types − Passive elements − store/use pwr.
− Active elements − supplies pwr.

Model − Basic circuit elements

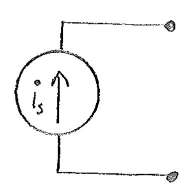
i) two terminals

ii) Cannot be divided to anything smaller

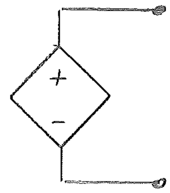
Active (Sources)



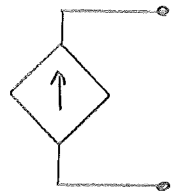
Independent Voltage Source



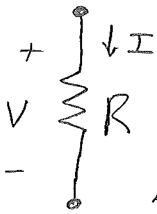
Independent current Source

$V_s = \mu V_1$ 

Dependent Voltage Source

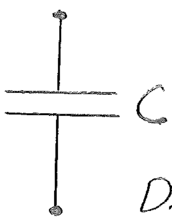
$i_s = \beta i_1$ 
Dependent Current Source.

Passive - elements



→ Resistor, units Ohms, (Ω).

Denoted: Resistance



→ Capacitor, units Farads, (F)

Denoted: Capacitance

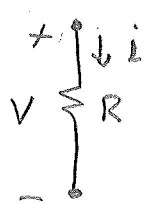
→ Inductor, units Henrys, (H).

Denoted: Inductance

Conductance is defined as $G = \frac{1}{R}$ (Siemens) or (S).

Ohm's Law

Consider:

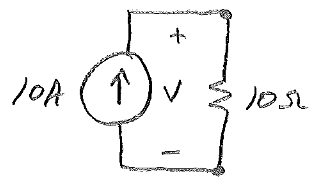


$\boxed{V = iR}$ - Note in this defn. +i is entering the +V term.

Ex1:

Consider

Find P_R

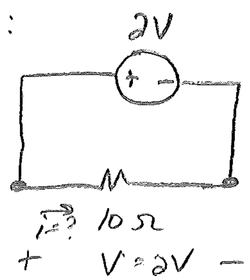


$P_R = Vi$. To find V we use $V = iR$ - ohm's law.

$\Rightarrow V = 10A \cdot 10\Omega = 100V. \Rightarrow P = 100V \cdot 10A = \boxed{1000W}$ or $\boxed{1\ KW.}$

Pwr. absorbed by R or supplied by the src.

Ex 2: Consider:



Find P_R.

From Ohm's law, $V = iR \Rightarrow i = \frac{2}{10} = .2A = 200mA$

$\Rightarrow P_R = 2V \cdot .2A = \boxed{.4W}$ or $\boxed{400mW}$

Kirchhoff's Laws

A circuit is said to be solved if every voltage across and the current through every element is known.

A node is defined to be the point where two or more circuit elements meet.

Kirchhoff's Current Law: The algebraic sum of all the currents at any node equal to zero,

or $$\sum_{m=1}^{M} (i_m)_{in} = \sum_{n=1}^{N} (i_n)_{out} .$$

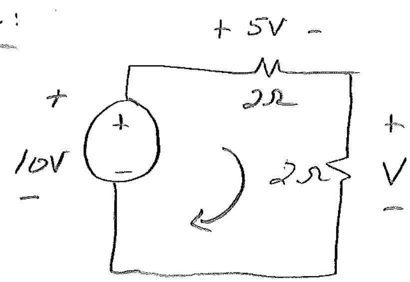
Short hand - $\underline{\underline{KCL}}$

Kirchhoff's Voltage Law: The algebraic sum around any closed path is equal to zero, or

$$\sum V_R = 0.$$

Short hand - $\underline{\underline{KVL}}$

Ex:



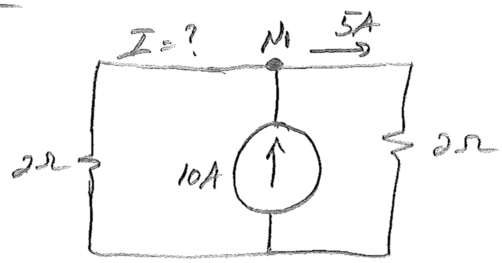
→ Applying KVL, define a dir. + sum voltages.

$$\Rightarrow \; -10V + 5V + V = 0$$

$$\Leftrightarrow \; \boxed{V = 5V}$$

Write the sign of the voltage to be the first sign we see around the loop.

Ex:



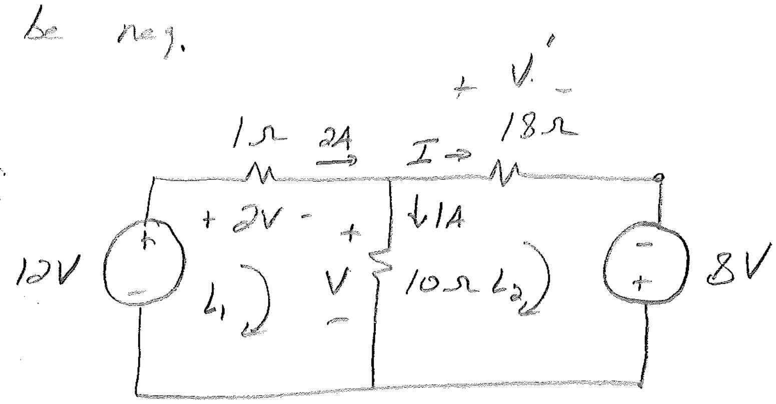
→ Apply KCL @ Node N_1,

$$\Rightarrow +10A - 5A - I = 0.$$

$$\Leftrightarrow \boxed{I = 5A}$$

Define currents entering a node to be pos. & leaving to be neg.

Ex:



1st way

$$I = 2A - 1A = \boxed{1A}$$

$$V = 12V - 2V = \boxed{10V}$$

Next, $L_2 \Rightarrow -8V - V + V' = 0$

$$\Leftrightarrow -8V - 10V = -V'$$

$$\Leftrightarrow \boxed{V' = 18V}$$

2nd way

Around L_1: $-12V + 2V + V = 0$

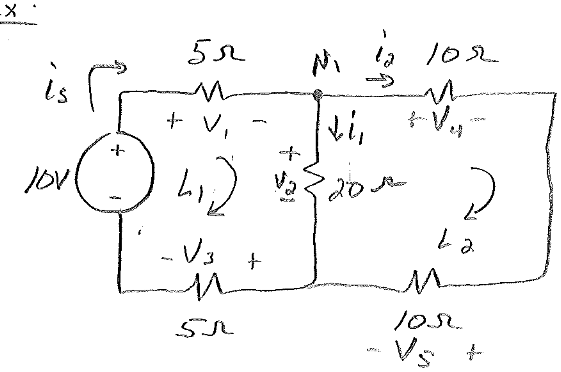
$$\Rightarrow \boxed{V = 10V}$$

Around L_2: $-8V - V + V' = 0$

$$\Leftrightarrow 8V + 10V = V'$$

$$\Leftrightarrow \boxed{V' = 18V}$$

Ex:



Determine $i_s + i_1$ if $V_1 = 2.5V$

KVL around L_1: $-10V + V_1 + V_2 + V_3 = 0$

$\Leftrightarrow V_1 + V_2 + V_3 = 10V$

From observation $V_1 = V_3 \Rightarrow 2V_1 + V_2 = 10V$

$\Rightarrow 5V + V_2 = 10$

$\Leftrightarrow V_2 = 5V.$

$\Leftrightarrow i_1 = \dfrac{V_2}{20} = \boxed{1/4\ A}$ or $\boxed{250mA}$

We also know @ N_1, $i_s = i_1 + i_2$

Need i_2. We know $V_2 = 5V$, then around L_2 we have:

$V_4 + V_5 = V_2$. Since the 10Ω res. share i_2, $V_4 = V_5$

$\Leftrightarrow 2V_4 = V_2 \Rightarrow V_4 = V_2/2 = \underline{2.5V} \Rightarrow i_2 = \dfrac{2.5V}{10} = .25A$

$= 250mA$

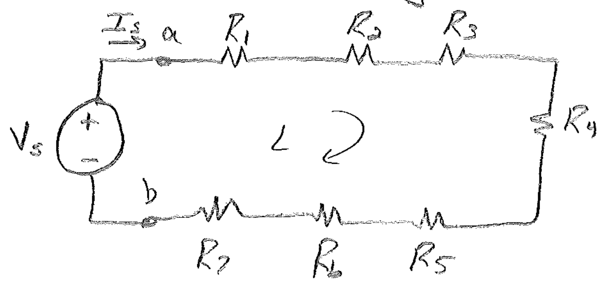
$\therefore i_s = i_1 + i_2$

$= 250mA + 250mA$

$= \boxed{500mA}$

Resistors in series

Consider the following circuit:



Series connected circuit b/c they share the same current.

where $R_1 \neq R_2 \neq R_3 \neq R_4 \neq R_5 \neq R_6 \neq R_7$

Applying KVL gives $-V_s + I_s R_1 + I_s R_2 + \ldots + I_s R_7$

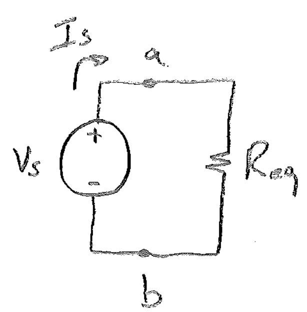
$$\Leftrightarrow V_s = I_s \underbrace{(R_1 + R_2 + \ldots + R_7)}_{=: R_{eq}}$$

$\therefore$ we define the equivalent resistance of the series conn. resistors as:

$$R_{eq} = R_1 + R_2 + \ldots + R_7.$$

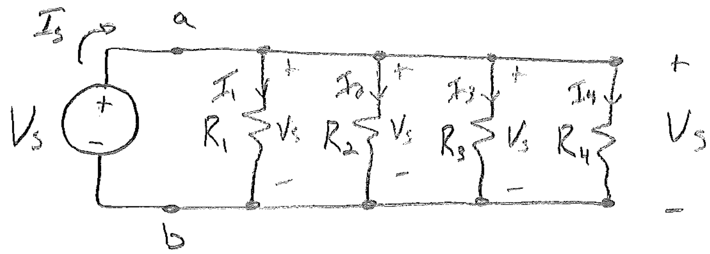
$\therefore$ in general: $\boxed{R_{eq} = \sum_{i=1}^{N} R_i}$ $\rightarrow$ Trade-off is info. is lost about the ckt. such as individual pwrs. + voltages

$\Rightarrow$ The above circuit can be written as:



Resistors in Parallel

Consider the following circuit:



Parallel conn. circuit b/c they share the same voltage.

where $R_1 \neq R_2 \neq R_3 \neq R_4$.

Apply KCL @ the top node gives

$$+I_s - I_1 - I_2 - I_3 - I_4 = 0$$

$$\Leftrightarrow \quad I_s = I_1 + I_2 + I_3 + I_4.$$

Ohm's law

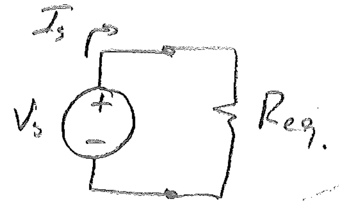
$$\Leftrightarrow \quad I_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \frac{V_s}{R_4}$$

$$\Leftrightarrow \quad I_s = V_s \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)}_{= \frac{1}{R_{eq}}}$$

$\therefore$ the equivalent resistance of N resistors conn. in parallel is:

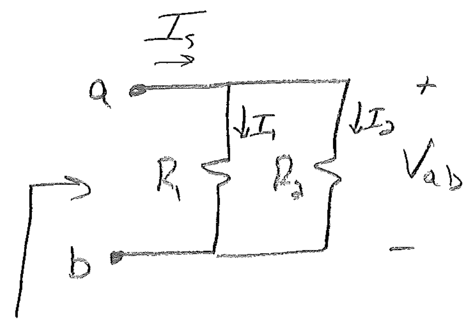
$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}}$$

Then the above ckt. can be written as:



2-resistors in parallel:

Common prob:



R_{eq}

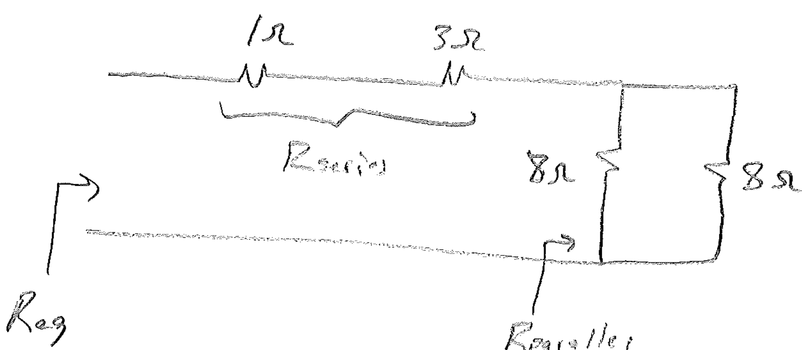
We can find R_{eq} w/ the following:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Leftrightarrow \quad \frac{R_1 R_2}{R_{eq}} = R_2 + R_1$$

$$\Leftrightarrow \quad \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Ex: Find R_{eq}:

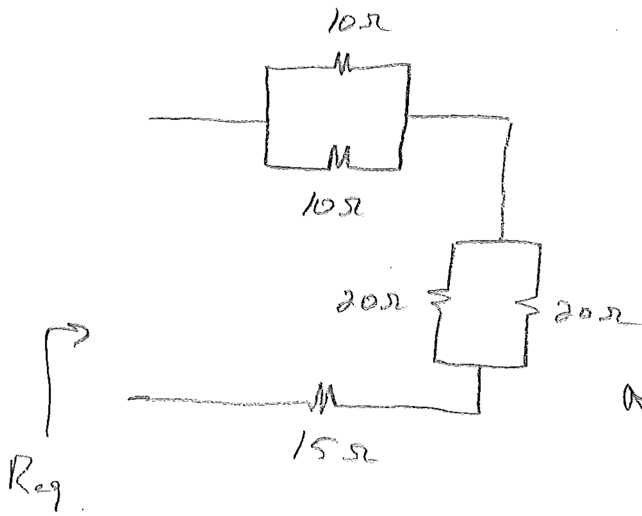


$$R_{eq} = R_{series} + R_{parallel}$$

$$= 1\Omega + 3\Omega + 8 \parallel 8$$

$$= 4 + \frac{64}{16} = \boxed{8\Omega}$$

Ex: Find R_{eq}:



$10 \parallel 10 \Rightarrow R_{eq,10} = \frac{10 \cdot 10}{20} = 5\Omega$

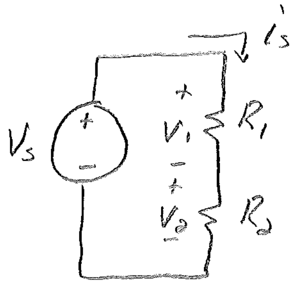
redraw w/ Req.

$20 \parallel 20 \Rightarrow R_{eq,20} = 10\Omega$

$\Rightarrow R_{eq} = (5 + 10 + 15)\Omega = \boxed{30\Omega}$

Voltage Division

Voltage division often refers to a 2-resistor ckt, as shown here:



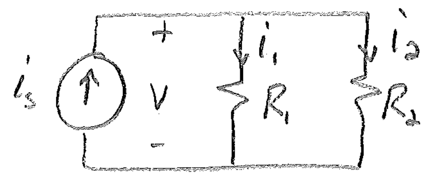
Say we want V_1 or V_2. Using KVL & ohm's law gives:

$$V_s = V_1 + V_2$$
$$= V_1 + i_s R_2$$
$$= V_1 + \frac{V_1}{R_1} R_2$$
$$= V_1 \left(\frac{R_1 + R_2}{R_1} \right)$$

$$\Rightarrow \boxed{V_1 = V_s \frac{R_1}{R_1 + R_2}}$$

Current Division

In a similar manner we can relate the branch currents in a 2-resistor network, as shown below,



KCL & Ohm's law gives:

$$i_s = i_1 + i_2$$

$$= i_1 + \frac{V}{R_2}$$

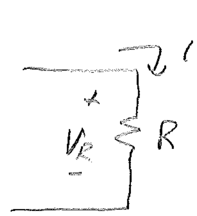
$$= i_1 + \frac{i_1 R_1}{R_2}$$

$$= i_1\left(\frac{R_2 + R_1}{R_2}\right)$$

$$\Rightarrow \boxed{i_1 = i_s \frac{R_2}{R_1 + R_2}}$$

Power Computations

Known i + R:



We know the power absorbed by R is $P_A = V_R i$.

We also know $V_R = iR$ (ohm's law). $\Rightarrow P_A = iR \cdot i$

$$= \boxed{i^2 R}$$

Known V & R:

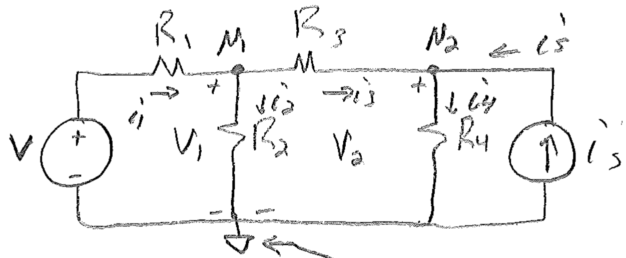
Again $P = V_R i$. we know $i = \frac{V_R}{R} \Rightarrow P = V_R i$

$$= V_R \frac{V_R}{R} = \boxed{V^2/R}$$

Nodal Analysis

Works for all circuits, + introduced w/ an example.

Consider:



Step 1: Define reference node. Typically here.

Step 2: Define nodes. Denoted as N_1 + N_2 above.

Step 3: Write nodal voltages w.r.t. the ref. node. Denoted as V_1 + V_2 above.

Step 4: Denote currents in ckt.

Step 5: Write node voltage eqns. + solve:

$$+\left(\frac{V - V_1}{R_1}\right) - \frac{V_1}{R_2} - \frac{V_1 - V_2}{R_3} = 0 \qquad ①$$

and

$$+\left(\frac{V_1 - V_2}{R_3}\right) - \frac{V_2}{R_4} + i_s = 0 \qquad ②$$

2 eqns. 2 unkns.

Now let $V = 10$, $R_1 = 1$, $R_2 = 5$, $R_3 = 2$, $R_4 = 10$ + $i_s = 2A$.

For ①: $\frac{10 - V_1}{1} - \frac{V_1}{5} - \frac{V_1 - V_2}{2} = 0$

For ②: $\frac{V_1 - V_2}{2} + 2 = \frac{V_2}{10}$

Solving for V_1 in ①:

$$10 - V_1 = \frac{V_1}{5} + \frac{V_1}{2} - \frac{V_2}{2}$$

$$\Leftrightarrow 10 - V_1 - \frac{V_1}{5} - \frac{V_1}{2} = -\frac{V_2}{2}$$

$$\Leftrightarrow 10 - V_1\left(1 + \frac{1}{5} + \frac{1}{2}\right) = -\frac{V_2}{2}$$

$$\Leftrightarrow 10 - \frac{17}{10}V_1 = -\frac{V_2}{2}$$

$$\Leftrightarrow -\frac{17}{10}V_1 = -\frac{V_2}{2} - 10$$

$$\Leftrightarrow V_1 = \frac{10}{17}\left(\frac{V_2}{2} + 10\right) \rightarrow \text{sub into } ②$$

$$\Rightarrow 10\left(\frac{\frac{10}{17}\left(\frac{V_2}{2}+10\right) - V_2}{2} + 2\right) = \left(\frac{V_2}{10}\right)10$$

$$\Leftrightarrow \frac{50}{17}\left(\frac{V_2}{2}+10\right) - 5V_2 + 20 = V_2$$

$$\Leftrightarrow \frac{25}{17}V_2 + \frac{500}{17} - 5V_2 + 20 = V_2$$

$$\Leftrightarrow V_2\left(\frac{25}{17} - 5 - 1\right) = -\frac{500}{17} - 20$$

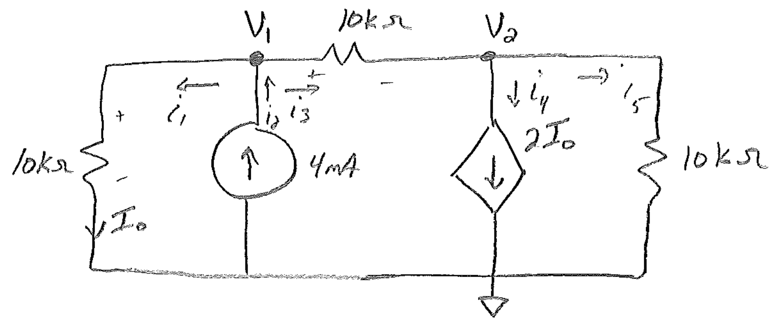
$$\Leftrightarrow \boxed{V_2 = 10.9\,V}$$

sub.

$$\Rightarrow V_1 = \frac{10}{17}\left(\frac{10.9}{2} + 10\right)$$

$$\Rightarrow \boxed{V_1 = 9.08\,V}$$

Ex: Find V_1 & V_2



$N_1: \; -i_1 + i_2 - i_3 = 0$

$\Rightarrow \; i_2 = \cancelto{I_0}{i_1} + i_3$

$\Rightarrow \; 4mA = \frac{V_1}{10k} + \frac{V_1 - V_2}{10k}$

solving for V_1

$4mA = V_1\left(\frac{1}{10k} + \frac{1}{10k}\right) - V_2\,\frac{1}{10k}$

$\Rightarrow \; V_1 = \dfrac{4mA + V_2\frac{1}{10k}}{2\times10^{-4}}$

$= 20 + \frac{1}{2}V_2$

$N_2: \; i_3 - i_4 - i_5 = 0$

$\Rightarrow \; i_3 = i_4 + i_5$

$\Rightarrow \; \frac{V_1 - V_2}{10k} = 2\cancelto{V_1/10k}{I_0} + \frac{V_2}{10k}$

sub

$\Rightarrow \; \frac{20 + \frac{1}{2}V_2}{\cancel{10k}} - \frac{V_2}{\cancel{10k}} = \left(20 + \frac{1}{2}V_2\right)\frac{2}{\cancel{10k}} + \frac{V_2}{\cancel{10k}}$

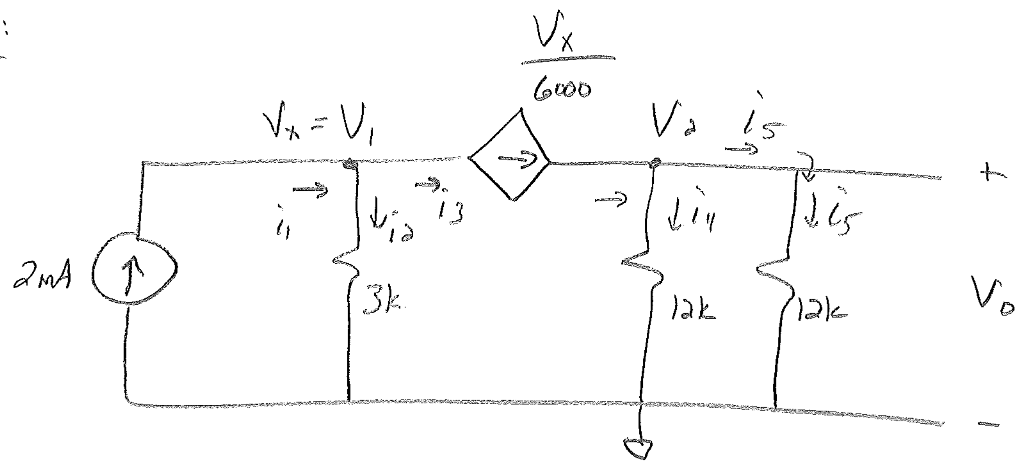
$\Rightarrow \; 20 + \frac{1}{2}V_2 - V_2 = 40 + V_2 + V_2$

$\Rightarrow \; 20 - 40 = V_2\left(\cancelto{5/2}{1 + 1 + 1 - \tfrac{1}{2}}\right)$

$\Rightarrow \; -20 \cdot \frac{2}{5} = V_2$

$\Rightarrow \; \boxed{V_2 = -8V} \;\Rightarrow\; V_1 = 20 + \frac{1}{2}(-8) = \boxed{16V}$

Ex:



N_1:

$i_1 = i_2 + i_3$

$\Rightarrow 2mA = \frac{V_1}{3k} + \frac{V_x}{6000} \quad \Big| \; V_1 = V_x$

$\Rightarrow 2mA = \frac{V_x}{3k} + \frac{V_x}{6k}$

$\Rightarrow V_x = 4$

N_2: Then $i_3 = i_4 + i_5$

$\Rightarrow \frac{4}{6000} = \frac{V_2}{12k} + \frac{V_2}{12k} \quad \Big| \; V_2 = V_o$

$\Rightarrow \frac{4}{6k} = V_o\left(\frac{1}{12k} + \frac{1}{12k}\right)$

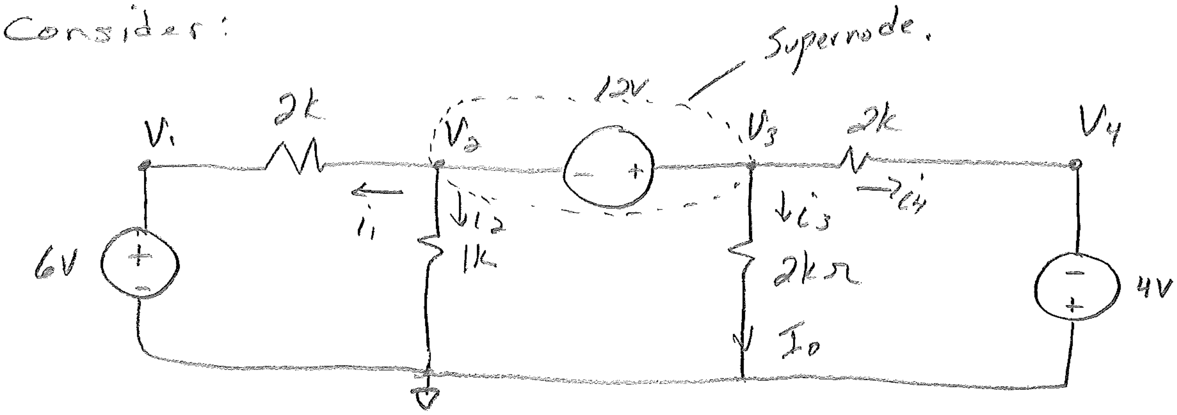
$\Rightarrow V_o = \frac{2 \cdot 4}{2}$

$= \boxed{4V}$

Supernodes

If a voltage exists between two nodes that are neither the reference node, we can define a supernode & sum eqns.

Consider:



Find I_o:

We know $V_1 = 6V$ + $V_4 = -4V$. Treat supernode as one node + sum currents:

$$-i_1 - i_2 - i_3 - i_4 = 0 \quad \text{sub}$$

$$i_1 = \frac{V_2 - V_1}{2k}, \quad i_2 = \frac{V_2}{1k}, \quad i_3 = \frac{V_3}{2k}, \quad i_4 = \frac{V_3 - V_4}{2k}$$

$$-\left(\frac{V_2 - V_1}{2k}\right) - \frac{V_2}{1k} - \frac{V_3}{2k} - \frac{V_3 - V_4}{2k} = 0 \Bigg|_{V_1 = 6V,\ V_4 = -4V}$$

$$\Rightarrow -\left(\frac{V_2 - 6}{2k}\right) - \frac{V_2}{1k} - \frac{V_3}{2k} - \frac{V_3 + 4V}{2k} = 0$$

Also $V_3 - V_2 = 12V$

$\Rightarrow V_3 = 12V + V_2$

sub

$$\Rightarrow -\left(\frac{V_2 - 6}{2k}\right) - \frac{V_2}{1k} - \frac{12 + V_2}{2k} - \frac{12 + V_2 + 4}{2k}$$

Solving for V_2:

$$-\frac{V_2}{2k} + \frac{6}{2k} - \frac{V_2}{1k} - \frac{12}{2k} - \frac{V_2}{2k} = \frac{12}{2k} + \frac{V_2}{2k} + \frac{4}{2k}$$

$$\Rightarrow V_2\left(-\frac{1}{2k} - \frac{1}{1k} - \frac{1}{2k} - \frac{1}{2k}\right) = -\frac{6}{2k} + \frac{12}{2k} + \frac{12}{2k} + \frac{4}{2k}$$

$$\Rightarrow V_2 = \frac{0.011}{-2.5\times10^{-3}}$$

$$= -4.4V$$

$$\Rightarrow V_3 = 12 - 4.4V = 7.6V$$

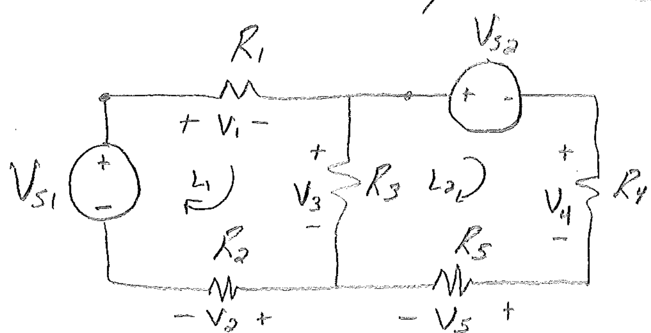
$$\Rightarrow I_o = \frac{V_3}{2k} = \boxed{3.8mA}$$

Mesh Analysis

Mesh analysis is another general technique (similar to nodal) based on KVL. Note that nodal is based on KCL.

Lets introduce the idea w/ an example.

Consider:



Step 1: Label all voltages

Step 2: Define loops $L_1, L_2, \ldots L_N$

Step 3: Defin loop currents $I_1, I_2, \ldots I_N$

Step 4: Write KVL eqns. around each loop.

Step 5: Write voltages in terms of loop currents

L1:

$$V_1 + V_3 + V_2 - V_{S1} = 0$$

L2:

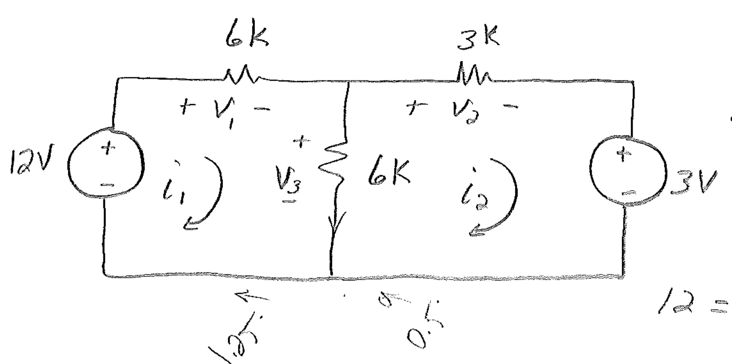
$$V_{S2} + V_4 + V_5 - V_3 = 0$$

From Ohms law

$$\underbrace{V_1 = i_1 R_1,\quad V_2 = i_1 R_2,\quad V_3 = (i_1 - i_2) R_3,\quad V_4 = i_2 R_4 + V_5 = i_2 R_5}_{\substack{\text{sub into L1 + L2} \\ \text{eqns.}}}$$

$$\Rightarrow \underbrace{i_1 R_1 + (i_1 - i_2) R_3 + i_1 R_2 = V_{S1} \quad \& \quad V_{S2} + i_2 R_4 + i_2 R_5 = V_3}_{\text{2 eqns. 2 unkn. \& solve.}}$$

Ex:



L1: $12V = i_1 6k + 6k(i_1 - i_2)$ ①

L2: $-3V = -6k(i_1 - i_2) + i_2 3k$ ②

Solving for i_1:

$$12 = i_1(6k + 6k) - 6k i_2$$

$$\Rightarrow \frac{12 + 6k i_2}{12k} = i_1$$

$\Rightarrow \; 1\times10^{-3} + 0.5 i_2 = i_1 \leftarrow$ sub into ③

$\Rightarrow \; -3V = -6k\left(1\times10^{-3} + 0.5 i_2 - i_2\right) + i_2 3k$

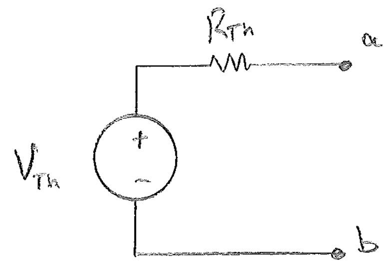
$\Rightarrow \; -3V = -6 - \cancel{3k i_2} + 6k i_2 + \cancel{i_2 3k}$

$\Rightarrow \; 3V = 6k i_2 \Rightarrow \boxed{i_2 = 0.5mA}$

$\Rightarrow \; i_1 = 1mA + 0.5 \cdot 0.5mA = \boxed{1.25mA}$

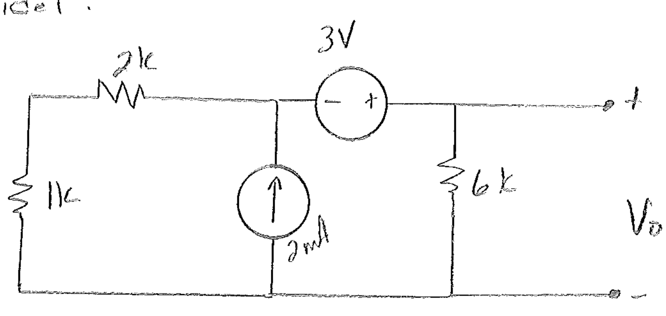
Thévenin + Norton Equivalents

Thévenin equivalent ckt: A series connected independent voltage source (V_{Th}) + resistance (R_{Th}) used to model a ckt connected between two terminals, say a + b, or



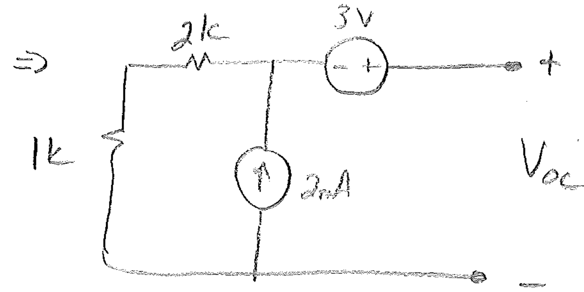
Intro through an example:

Consider:



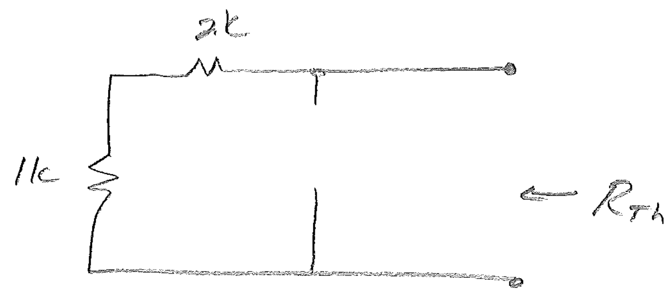
For circuits w/ only independent sources:

1) Remove the resistor w/ V_o & compute V_{oc}.



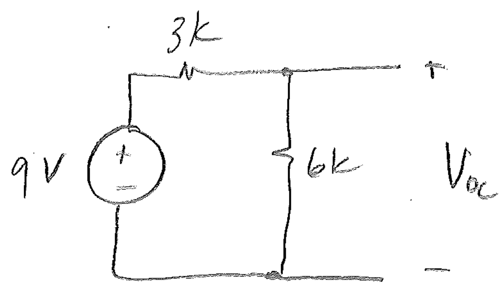
$V_{oc} = 2mA \cdot 3k + 3V = 9V$

2) Zero-out the sources by short circuiting the voltages & open circuiting the current sources:



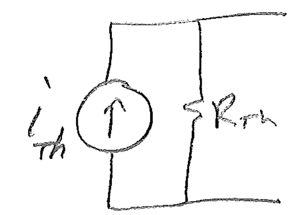
$R_{Th} = 2k + 1k = 3k$

3) Redraw ckt. attached to resistor of interest:



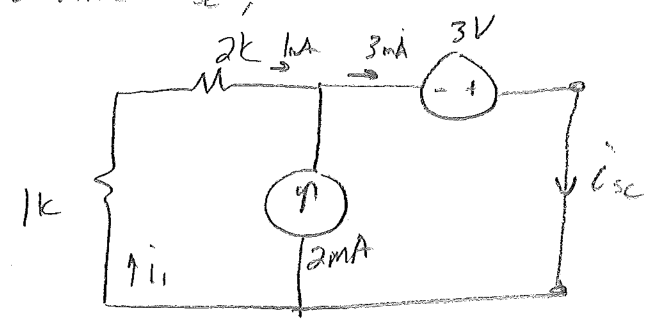
$\Rightarrow V_{oc} = 9V \cdot \frac{6k}{9k} = \boxed{6V}$

Norton equivalent ckt: Similar to Thevenin, except R_{Th} is connected in parallel w/ i_{Th}, or a current source,



For ckts. w/ only independent srcs.

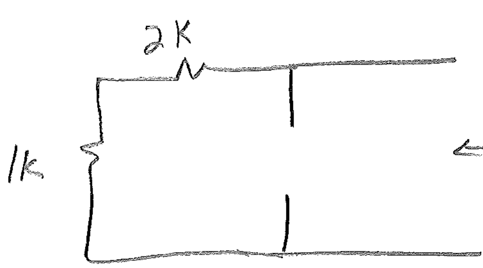
1) Remove the resistor w/ V_o + short the terminals to find i_{sc}, or



$$i_1 = \frac{3V}{1k + 2k} = 1mA \Rightarrow i_{sc} = 3mA$$

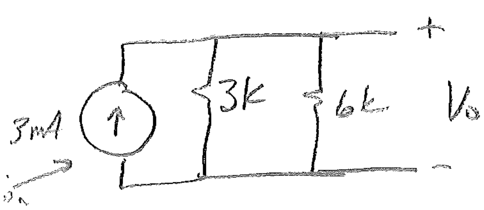
2) Same as Thevenin ckt. process to find R_{Th}.

$\Rightarrow$



$\leftarrow R_{Th} = 3k$

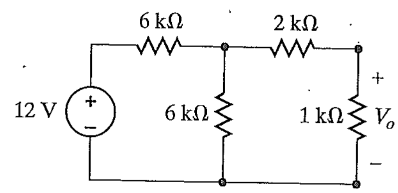
3) Draw Norton equiv:



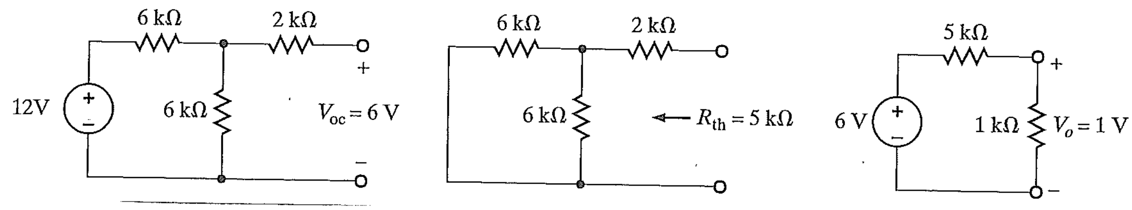
Note direction

Ex1 :

D4.5 Find V_o in the following network using Thévenin's theorem.



ANSWER:

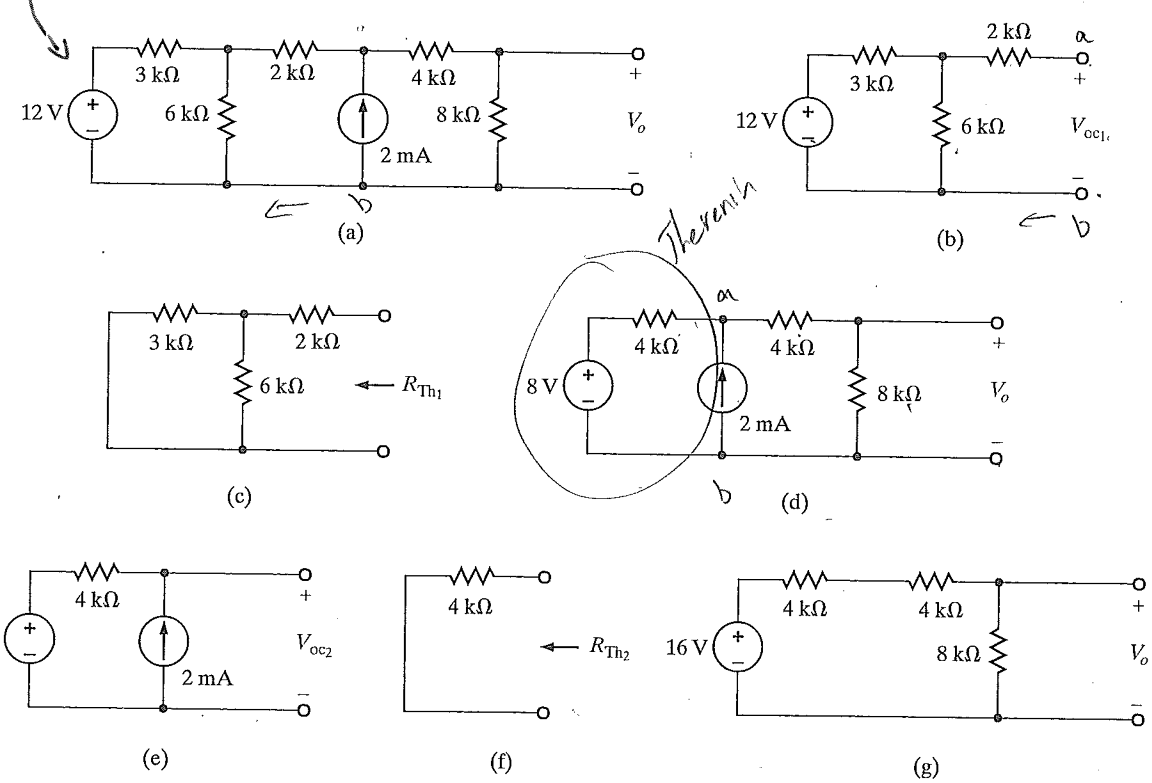


Ex 2:

Let us use Thévenin's theorem to find V_o in the network in Fig. 4.8a, which is redrawn in Fig. 4.12a.

here

SOLUTION If we break the network to the left of the current source, the open-circuit voltage V_{oc_1} is as shown in Fig. 4.12b. Since there is no current in the 2-kΩ resistor and therefore no voltage across it, V_{oc_1} is equal to the voltage across the 6-kΩ resistor, which can be determined by voltage division as



$$V_{oc_1} = 12\left(\frac{6k}{6k + 3k}\right) = 8 \text{ V}$$

The Thévenin equivalent resistance, R_{Th_1}, is found from Fig. 4.12c as

$$R_{Th_1} = 2k + \frac{(3k)(6k)}{3k + 6k} = 4 \text{ k}\Omega$$

Connecting this Thévenin equivalent back to the original network produces the circuit shown in Fig. 4.12d. We can now apply Thévenin's theorem again, and this time we break the network to the right of the current source as shown in Fig. 4.12e. In this case V_{oc_2} is

$$V_{oc_2} = (2 \times 10^{-3})(4k) + 8 = 16 \text{ V}$$

and R_{Th_2} obtained from Fig. 4.12f is 4 kΩ. Connecting this Thévenin equivalent to the remainder of the network produces the circuit shown in Fig. 4.12g. Simple voltage division applied to this final network yields $V_o = 8$ V. Norton's theorem can be applied in a similar manner to solve this network; however, we save that solution as an exercise for the reader. ❑

Circuits with both Dependent & Independent Sources:

For these cases, we need to compute V_{oc} & I_{sc}.

Then $R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

Note that we need to choose a point to remove the resistor of interest such that any dependent voltage or currents remain in the ckt.

Lets look at an example:

EXAMPLE 4.12

Let us use Thévenin's theorem to find V_o in the network in Fig. 4.16a.

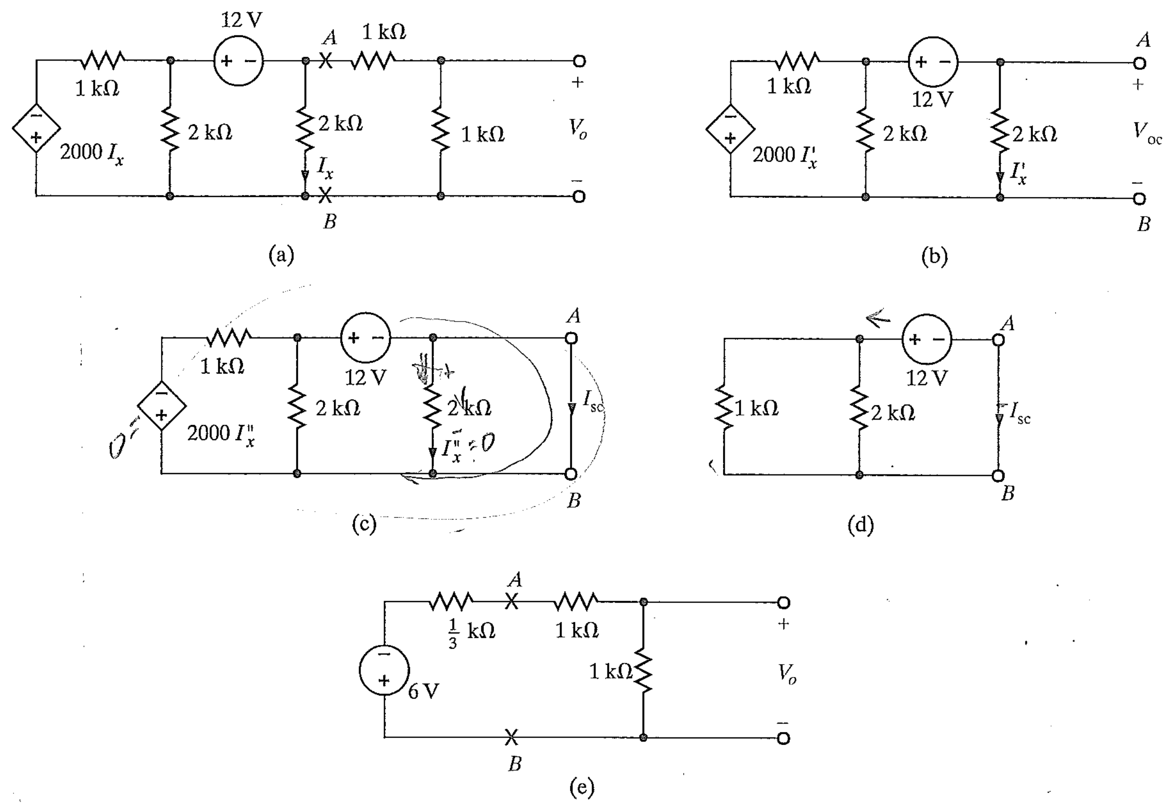


Figure 4.16 Circuits used in Example 4.12.

SOLUTION To begin, we break the network at points *A-B*. Could we break it ju to the right of the 12-V source? No! Why? The open-circuit voltage is calculat from the network in Fig. 4.16b. Note that we now use the source $2000I_x'$ becau this circuit is different from that in Fig. 4.16a. KCL for the supernode around the 12-V source is

$$\frac{(V_{oc} + 12) - (-2000I'_x)}{1k} + \frac{V_{oc} + 12}{2k} + \frac{V_{oc}}{2k} = 0$$

where

$$I'_x = \frac{V_{oc}}{2k}$$

yielding $V_{oc} = -6$ V.

I_{sc} can be calculated from the circuit in Fig. 4.16c. Note that the presence of the short circuit forces I''_x to zero and, therefore, the network is reduced to that shown in Fig. 4.16d.

Therefore,

$$I_{sc} = \frac{-12}{\frac{2}{3}k} = -18 \text{ mA}$$

Then

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{1}{3} k\Omega$$

Connecting the Thévenin equivalent circuit to the remainder of the network at terminals A-B produces the circuit in Fig. 4.16e. At this point, simple voltage division yields

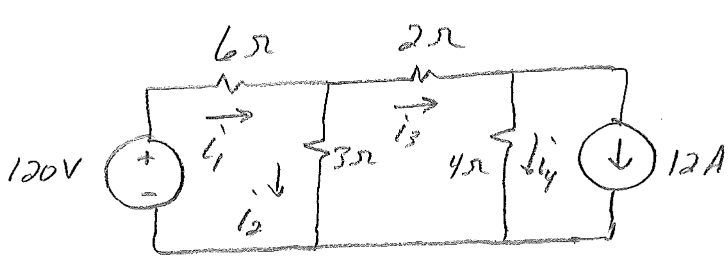
$$V_o = (-6)\left(\frac{1k}{1k + 1k + \frac{1}{3}k}\right) = \frac{-18}{7} \text{ V}$$

❑

Super Position

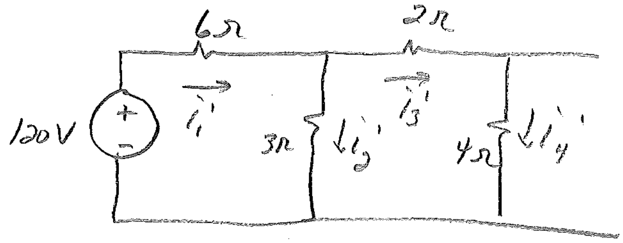
For a given ckt. w/ only independent sources, we can apply each src. individually & add the results of each of the sources for the total response.

Consider:



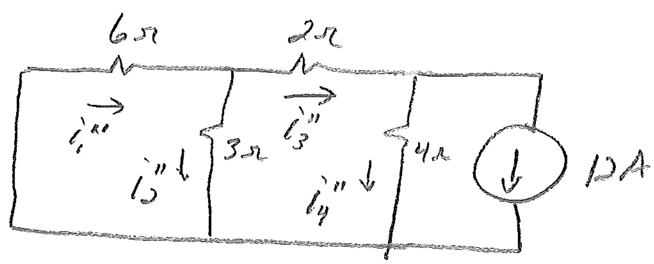
Step 1: Turn off all sources except the 1^{st} one.

Step 2: Draw ckt. + solve.



It can be shown $i_1' = 15A$, $i_2' = 10A$ + $i_3' = 5A = i_4'$

Step 3: Repeat steps 1 + 2 ∀ sources.



$\Rightarrow i_1'' = 2A$, $i_2'' = -4A$, $i_3'' = 6A$ + $i_4'' = -6A$

$\therefore$ $i_1 = i_1' + i_1'' = 17A$, $i_2 = i_2' + i_2'' = 6A$

$i_3 = i_3' + i_3'' = 11A$, $i_4 = i_4' + i_4'' = -1A$