MATH 724
FALL 2010
HOMEWORK 4

Due Monday, April 6, 2010

1. (5 pt) Show that if R is Noetherian and ω is almost integral over R then ω is integral over R.

2. Let $F \subseteq K$ be fields.
   a) (5 pt) Show that $F + xK[x]$ is an HFD.
   b) (5 pt) Show that $(F + xK[x])[t]$ is an HFD if and only if F is algebraically closed in K.
   c) (5 pt) It is known that if $R[x]$ is an HFD, then R must be integrally closed. Explain why completely integrally closed is not necessary.

3. Give examples of domains with the following properties.
   a) (5 pt) An atomic domain with nonatomic integral closure.
   b) (5 pt) A nonatomic domain (with at least one irreducible) where everything that can be factored into irreducibles, factors uniquely.
   c) (5 pt) A domain (not a field) that contains no atoms whatsoever.
   d) (5 pt) A nonatomic domain where every irreducible is prime.