MATH 724
FALL 2010
HOMEWORK 5

Due Monday, April 23, 2010

1. (5 pt) We have (or will have) shown that if $R[x]$ is an HFD, then R must be integrally closed. Can we replace "integrally closed" with "completely integrally closed"?

2. We say that a domain, R, is an AP-domain if every atom is prime.
   a) (5 pt) Show that any GCD-domain is an AP-domain.
   b) (5 pt) Show that in the class of atomic domains, the notions of AP-domain, GCD-domain, and UFD are all equivalent.

3. We will (unfortunately) say that R is a U-UFD if every nonzero nonunit that can be factored into irreducibles does so uniquely.
   a) (5 pt) Show that any AP-domain is a U-UFD.
   b) (5 pt) Show that if R is a domain with precisely one irreducible and this irreducible is not prime, then R is a U-UFD that is not an AP-domain.

4. We say that R is a CK-domain if R is atomic and has only finitely many irreducibles (up to associates). We use the notation CK-n to refer to a CK-domain with precisely n irreducibles.
   a) (5 pt) Show that if R is CK-n for $n \leq 2$ then R is a PID (and hence a UFD).
   b) (5 pt) Find an example of a CK-3 domain that is not a UFD.