MATH 724
FALL 2010
HOMEWORK 6

Due Monday, May 10, 2010

1. Let R be a very strongly atomic ring and $e \in R$ an idempotent.

   a) (5 pt) Show that e is either 0 or 1.

   b) (5 pt) Use this to find all values of $n > 1$ such that $\mathbb{Z}/n\mathbb{Z}$ is very strongly atomic.

2. Let R be an atomic domain and $x \in R$ a nonzero, nonunit. Let $\mathrm{Irr}(R)$ denote the set of distinct (up to associates) irreducible divisors of x. We define the irreducible divisor graph of the element x (G_x) to be the graph with vertices from $\mathrm{Irr}(R)$ and we declare that if $u, v \in \mathrm{Irr}(R)$ we say that u and v have an edge between them if uv divides x (we ignore the possibility of loops). Prove the following conditions are equivalent.

   a) R is a UFD.

   b) G_x is connected for all nonzero, nonunits $x \in R$.

   c) G_x is complete for all nonzero, nonunits $x \in R$.