Due Friday, October 27, 2006.

1. Let R be a Dedekind domain. Find all possible ways to construct irreducibles given the class groups listed below. Use this to bound the elasticity (what assumptions do you need so the the elasticity bound is obtained)?

    a) (5 pt) $\mathrm{Cl}(R) \cong \mathbb{Z}/4\mathbb{Z}$.

    b) (5 pt) $\mathrm{Cl}(R) \cong \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$.

    c) (5 pt) $\mathrm{Cl}(R) \cong \mathbb{Z}/6\mathbb{Z}$.

2. (5 pt) Construct an atomic domain, R which has non-atomic integral closure.

3. Let $d < 0$ be a square-free integer. We define

$$\omega = \begin{cases} \sqrt{d} & \text{if } d \equiv 2,3 \bmod(4), \\ \frac{1+\sqrt{d}}{2} & \text{if } d \equiv 1 \bmod(4) \end{cases}$$

and we consider the ring $R := \mathbb{Z}[\omega]$.

    a) (5 pt) Show that if R is a UFD then d must be prime (or $d = -1$).

    b) (5 pt) Show that if R is an HFD then d must be of the form $-p$ or $-pq$ where $p, q > 0$ are prime integers.

    c) (5 pt) Determine the status of the converse of parts a) and b).