

MATH 725
FALL 2006
HOMEWORK 5

Due Wednesday, November 29, 2006.

1. (5 pt) Let $F \subseteq K$ be fields. Show that $(F + K[x])[t]$ is an HFD if and only if F is algebraically closed in K .
2. (5 pt) We have seen that for $R[x]$ to be an HFD, then R must be an integrally closed HFD. Is it true that R must be a completely integrally closed HFD (this is certainly true if R is Noetherian)? Prove the statement or give a counterexample.
3. We say that the domain R is an AP-domain if all atoms (irreducibles) in R are prime.
 - a) (5 pt) Show that any GCD-domain is an AP-domain.
 - b) (5 pt) Show that if R is atomic then the following are equivalent:
 - (1) R is a GCD-domain.
 - (2) R is an AP-domain.
 - (3) R is a UFD.
 - c) (5 pt) Show that if R is a GCD-domain, then $R[x]$ is a GCD-domain.
 - d) (5 pt) Does c) hold for power series extensions?
 - e) (5 pt) Are the notions of GCD-domain and AP-domain equivalent?