MATH-266
Homework #8 Complex Review
Due 07/03/14 **Name:** _____

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

For any given complex number z, let $\arg(z)$ denote the argument of z.

1. Let $z = 1 + i$ and $w = -2 + 2\sqrt{3}i$. Compute the following:
   (a) $z + w$
   (b) zw
   (c) $\arg(z)$
   (d) $\arg(w)$
   (e) $\arg(zw)$
   (f) $\overline{z}$
   (g) $|z|$
   (h) $|w|$
   (i) $\dfrac{z}{w}$
   (j) $\dfrac{z\overline{w}}{|w|^2}$
2. Use Euler's formula and the identity $e^{x+y} = e^x e^y$ to write each of the following in the form $a + bi$:
   (a) $e^{1+\frac{\pi}{2}i}$
   (b) $e^{-\pi i}$
3. (a) Let θ be any rel number, and set $z = e^{i\theta}$. Show that $|z|$=1.
   (b) Let $z = a + bi$ be a complex number. Show that $|e^z| = e^a$.
4. Find all complex numbers z for which $z^2 = \overline{z}$.
5. Show that for any complex number x,
   (a) $\cos(x) = \dfrac{e^{ix} + e^{-ix}}{2}$ and
   (b) $\sin(x) = \dfrac{e^{ix} - e^{-ix}}{2i}$