Handout: Sketch of Strategies for Solving First-Order ODEs

Separable Equations: *Separate and integrate.*
Form: $g(y)y' = f(t)$

(1) Put the equation in the form above, then integrate both sides with respect to t for an implicit solution.
(2) Solve for y if possible.

Linear Equations: *Integrating factors.*
Form: $y' + p(t)y = q(t)$

(1) Choose an integrating factor $\mu(t)$ from the class of functions $e^{\int p(t)dt}$. Multiply on both sides of the equation by μ.
(2) The left-hand side now looks like $\frac{d}{dt}(\mu y)$. Integrate both sides with respect to t for an implicit solution.
(3) Solve for y if possible.

Exact Equations: *Find a potential.*
Form: $M(t,y) + N(t,y)y' = 0$ where $M = f_t$ and $N = f_y$ for some differentiable function $f(t,y)$.

(1) Test for exactness if necessary: check whether or not $M_y = N_t$.
(2) Find a potential function f, which is a member of both of the classes of functions $\int M dt$ and $\int N dy$.
(3) The left-hand side now looks like $\frac{d}{dt}f(t,y)$. Integrate both sides with respect to t for an implicit solution.
(4) Solve for y if possible.

Bernoulli Equations: *Subsitute and reduce to the linear case.*
Form: $y' + p(t)y = q(t)y^n$

(1) Multiply both sides of the equation by $(1-n)y^{-n}$.
(2) Substitute $w = y^{1-n}$.
(3) The equation is now linear in w and t, so solve as in the linear case.
(4) Un-substitute and solve for y if possible.

Homogeneous Equations: *Substitute and reduce to the separable case.*
Form: $M(t,y) + N(t,y)y' = 0$ where M and N are homogeneous.

(1) Substitute $y = ut$ or $t = vy$.
(2) The equation is now separable in u and t (or in y and v), so solve as in the separable case.
(3) Un-substitute and solve for y if possible.