HW #12

1) a) $y''-y=2t+4$

$\lambda^2-1=0 \Rightarrow \lambda=\pm 1$

$\boxed{y_h=c_1e^{-t}+c_2e^t}$

$W(y_1,y_2)=\begin{vmatrix} e^{-t} & e^t \\ -e^{-t} & e^t \end{vmatrix}$

$=e^{-t}e^t+e^{-t}e^t=2$

$u_1(t)=\int -\frac{e^t(2t+4)}{2}\,dt$

$=\int(-te^t-2e^t)\,dt$

$=\int -te^t\,dt-2e^t$

$u=-t \quad dv=e^t$
$du=-dt \quad v=e^t$

$=-te^t-\int -e^t\,dt-2e^t$

$=-te^t+e^t-2e^t$

$=-te^t-e^t$

$u_2(t)=\int \frac{e^{-t}(2t+4)}{2}\,dt$

$=\int(te^{-t}+2e^{-t})\,dt$

$=\int te^{-t}\,dt-2e^{-t}$

$u=t \quad dv=e^{-t}$
$du=dt \quad v=-e^{-t}$

$=-te^{-t}-\int -e^{-t}\,dt-2e^{-t}$

$=-te^{-t}-e^{-t}-2e^{-t}$

$=-te^{-t}-3e^{-t}$

$$\boxed{y=c_1e^{-t}+c_2e^t+(-te^t-e^t)e^{-t}+(-te^{-t}-3e^{-t})e^t}$$

or $y=c_1e^{-t}+c_2e^t-t-1-t-3 \Longrightarrow \boxed{y=c_1e^{-t}+c_2e^t-2t-4}$

b) $y''+4y'+4y=e^{-2t}\ln t$

$\lambda^2+4\lambda+4=0 \Rightarrow (\lambda+2)^2=0 \Rightarrow \lambda=-2$

$\boxed{y_h=c_1e^{-2t}+c_2te^{-2t}}$

$W(y_1,y_2)=\begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t}-2te^{-2t} \end{vmatrix}$

$=e^{-4t}-2te^{-4t}+2te^{-4t}$

$=e^{-4t}$

$u_1(t)=\int -\frac{te^{-2t}(e^{-2t}\ln t)}{e^{-4t}}\,dt$

$=\int -t\ln t\,dt$

$u=\ln t \quad dv=t\,dt$
$du=\frac{1}{t}dt \quad v=\frac{t^2}{2}$

$=-\left(\frac{t^2}{2}\ln t-\int \frac{t^2}{2}\cdot\frac{1}{t}\,dt\right)$

$=-\frac{t^2}{2}\ln t+\frac{t^2}{4}$

$u_2(t)=\int \frac{e^{-2t}(e^{-2t}\ln t)}{e^{-4t}}\,dt$

$=\int \ln t\,dt$

$u=\ln t \quad dv=dt$
$du=\frac{1}{t}dt \quad v=t$

$=t\ln t-\int t\cdot\frac{1}{t}\,dt$

$=t\ln t-t$

$$\boxed{y=c_1e^{-2t}+c_2te^{-2t}+\left(-\frac{t^2}{2}\ln t+\frac{t^2}{4}\right)e^{-2t}+(t\ln t-t)te^{-2t}}$$

c) $y'' + y = 3\sec(t) - t^2 + 1$

$\lambda^2 + 1 \Rightarrow \lambda = \pm i$

$\boxed{y_h = c_1 \cos t + c_2 \sin t}$

$W(y_1, y_2) = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = 1$

$u_1(t) = \int \frac{-\sin t\,(3\sec t - t^2 + 1)}{1}\,dt$

$= \int \left(-\frac{3\sin t}{\cos t} + t^2 \sin t - \sin t\right) dt$

$= -3\int \frac{\sin t}{\cos t}\,dt + \int t^2 \sin t\,dt + \cos t$

$u = \cos t \quad du = -\sin t\,dt \qquad u = t^2 \quad dv = \sin t\,dt \quad du = 2t\,dt \quad v = -\cos t$

$= 3\int \frac{1}{u}\,du - t^2 \cos t - \int -2t\cos t\,dt + \cos t$

$= 3\ln|\cos t| - t^2\cos t + \int 2t\cos t\,dt + \cos t$

$u = 2t \quad dv = \cos t\,dt \quad du = 2dt \quad v = \sin t$

$= 3\ln|\cos t| - t^2\cos t + 2t\sin t - \int 2\sin t\,dt + \cos t$

$= 3\ln|\cos t| - t^2\cos t + 2t\sin t + 3\cos t$

$u_2(t) = \int \frac{\cos t\,(3\sec t - t^2 + 1)}{1}\,dt$

$= \int (3 - t^2\cos t + \cos t)\,dt$

$= 3t - \int t^2\cos t\,dt + \sin t$

$u = t^2 \quad dv = \cos t\,dt \quad du = 2t \quad v = \sin t$

$= 3t + \sin t - \left(t^2 \sin t - \int 2t\sin t\,dt\right)$

$u = 2t \quad dv = \sin t \quad du = 2dt \quad v = -\cos t$

$= 3t + \sin t - \left(t^2\sin t - \left(-2t\cos t - \int -2\cos t\,dt\right)\right)$

$= 3t + \sin t - t^2\sin t - 2t\cos t + 2\sin t\,dt$

$= 3t + 3\sin t - t^2\sin t - 2t\cos t$

$$\boxed{\begin{aligned} y = {} & c_1\cos t + c_2 \sin t \\ & + \cos t\,(3\ln|\cos t| - t^2\cos t + 2t\sin t + 3\cos t) \\ & + \sin t\,(3t + 3\sin t - t^2 \sin t - 2t\cos t) \end{aligned}}$$