HW #15

ⓐ $\mathcal{L}^{-1}\left\{\frac{1}{s^2+4s+8}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+4}\right\}(t)$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s+2)^2+4}\right\}(t)$

$= \boxed{\frac{1}{2}e^{-2t}\sin(2t)}$

ⓑ $\mathcal{L}^{-1}\left\{\frac{3s-15}{2s^2-4s+10}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{3s-15}{2(s^2-2s+5)}\right\}(t)$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3s-15}{(s-1)^2+4}\right\}(t)$

$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{3(s-1)-12}{(s-1)^2+4}\right\}(t)$

$= \frac{1}{2}\left[3\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\}(t) - 12\mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+4}\right\}(t)\right]$

$= \frac{3}{2}e^t\cos(2t) - \frac{12}{2}\cdot\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+4}\right\}(t)$

$= \boxed{\frac{3}{2}e^t\cos(2t) - 3e^t\sin(2t)}$

ⓒ $\mathcal{L}^{-1}\left\{\frac{6s^2-13s+2}{s(s-1)(s-6)}\right\}(t)$

$\frac{6s^2-13s+2}{s(s-1)(s-6)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-6}$

$\Rightarrow 6s^2-13s+2 = A(s-1)(s-6) + Bs(s-6) + Cs(s-1)$

$s=0: \quad 2 = A(-1)(-6) \Longrightarrow A = 1/3$

$s=1: \quad -5 = B(-5) \longrightarrow B = 1$

$s=6: \quad 216-78+2 = C(6)(5)$

$140 = 30C \Longrightarrow C = 14/3$

$= \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}(t) + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}(t) + \frac{14}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\}(t)$

$= \boxed{\frac{1}{3} + e^t + \frac{14}{3}e^{6t}}$

ⓓ $\mathcal{L}^{-1}\left\{\frac{s+11}{(s-1)(s+3)}\right\}(t)$

$\frac{s+11}{(s-1)(s+3)} = \frac{A}{s-1} + \frac{B}{s+3}$

$\Rightarrow s+11 = A(s+3) + B(s-1)$

$s=1: \quad 12 = 4A \longrightarrow A = 3$

$s=-3: \quad 8 = -4B \longrightarrow B = -2$

$= 3\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}(t) - 2\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}(t)$

$= \boxed{3e^t - 2e^{-3t}}$

ⓔ $\mathcal{L}^{-1}\left\{\frac{5s^2+34s+53}{(s+3)^2(s+1)}\right\}(t)$

$\frac{5s^2+34s+53}{(s+3)^2(s+1)} = \frac{A}{s+3} + \frac{B}{(s+3)^2} + \frac{C}{s+1}$

$\Rightarrow 5s^2+34s+53 = A(s+3)(s+1) + B(s+1) + C(s+3)^2$

$s=-1: \quad 24 = 4C \longrightarrow C = 6$

$s=-3: \quad -4 = -2B \longrightarrow B = 2$

$s=0: \quad 53 = 3A + B + 9C$

$53 = 3A + 2 + 9(6)$

$3A = -3 \longrightarrow A = -1$

$= -\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}(t) + 2\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2}\right\}(t) + 6\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}(t)$

$= \boxed{-e^{-3t} + 2te^{-3t} + 6e^{-t}}$

ⓕ $\mathcal{L}^{-1}\left\{\frac{7s^2-41s+84}{(s-1)(s^2-4s+13)}\right\}(t)$

$\Rightarrow \frac{7s^2-41s+84}{(s-1)(s^2-4s+13)} = \frac{A}{s-1} + \frac{Bs+C}{s^2-4s+13}$

$\Rightarrow 7s^2-41s+84 = A(s^2-4s+13) + (Bs+C)(s-1)$

$s=1: \quad 50 = 10A \longrightarrow A = 5$

$s=0: \quad 84 = 13A - C$

$84 = 13(5) - C \longrightarrow C = -19$

$s=-1: \quad 132 = 18A + (-B+C)(-2)$

$132 = 18(5) + (-B-19)(-2)$

$4 = 2B \longrightarrow B = 2$

$= 5\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{2s-19}{s^2-4s+13}\right\}$

$= 5e^t + \mathcal{L}^{-1}\left\{\frac{2(s-2)-15}{(s-2)^2+9}\right\}$

$= 5e^t + 2\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+9}\right\} - 5\mathcal{L}^{-1}\left\{\frac{3}{(s-2)^2+9}\right\}$

$= \boxed{5e^t + 2e^{2t}\cos(3t) - 5e^{2t}\sin(3t)}$