Section 1.1 # 2–10 even, 24, 26, [redacted] 34, 36, 38, 40, 60, 64

KEY

2) $y\dfrac{dy}{dt} + y^4 = \sin x$

$\boxed{\text{ODE}}$ $\boxed{\text{degree 1}}$ $\boxed{\text{Nonlinear}}$

4) $y''' - 2y'' + 5y' + y = e^x$

$\boxed{\text{ODE}}$ $\boxed{\text{degree 3}}$ $\boxed{\text{Linear}}$

6) $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = 0$

$\boxed{\text{ODE}}$ $\boxed{\text{degree 2}}$ $\boxed{\text{Linear}}$

8) $uu_x + u_t = 0$, $u = u(t, x)$

$\boxed{\text{PDE}}$

10) $\dfrac{d^2x}{dt^2} + 2\sin x = \sin 2t$

$\boxed{\text{ODE}}$ $\boxed{\text{degree 2}}$ ~~Linear~~ $\boxed{\text{Nonlinear}}$

24) $y'' - 12y' + 40y = 0$, $y_1 = e^{6x}\cos 2x$, $y_2 = e^{6x}\sin 2x$

$y_1' = 6e^{6x}\cos 2x - 2e^{6x}\sin 2x$

$y_1'' = 36e^{6x}\cos 2x - 12e^{6x}\sin 2x - 12e^{6x}\sin 2x - 4e^{6x}\cos 2x = 32e^{6x}\cos 2x - 24e^{6x}\sin 2x$

$y_1'' - 2y_1' + 40y = 32e^{6x}\cos 2x - 24e^{6x}\sin 2x - 72e^{6x}\cos 2x + 24e^{6x}\sin 2x + 40e^{6x}\cos 2x = 0$ ✓

$y_2' = 6e^{6x}\sin 2x + 2e^{6x}\cos 2x$

$y_2'' = 36e^{6x}\sin 2x + 12e^{6x}\cos 2x + 12e^{6x}\cos 2x - 4e^{6x}\sin 2x = 32e^{6x}\sin 2x + 24e^{6x}\cos 2x$

$y_2'' - 12y_2' + 40y_2 = 32e^{6x}\sin 2x + 24e^{6x}\cos 2x - 72e^{6x}\sin 2x - 24e^{6x}\cos 2x + 40e^{6x}\sin 2x = 0$ ✓

$\therefore y_1$ & y_2 are solutions to the given diff. eqn.

26) $y''' - 2y'' = 0$, $y = A + Bt + Ce^{2t}$

$y' = B + 2Ce^{2t}$

$y'' = 4Ce^{2t}$

$y''' = 8Ce^{2t}$

$y''' - 2y'' = 8Ce^{2t} - 8Ce^{2t} = 0$ ✓

$\therefore y$ is a solution to the given diff. eqn.

34) $\frac{dy}{dx} = (x^2-1)(x^3-3x)^3$

$y = \int (x^2-1)(x^3-3x)^3 = \int \frac{1}{3} u^3 du = \frac{1}{3}\frac{u^4}{4} + C = \boxed{\frac{1}{12}(x^3-3x)^4 + C}$

$u = x^3 - 3x$

$du = (3x^2-3)dx = 3(x^2-1)dx$

36) $\frac{dy}{dx} = \frac{x}{\sqrt{x^2-16}}$

$y = \int \frac{x}{\sqrt{x^2-16}} dx = \frac{1}{2}\int \frac{1}{\sqrt{u}} du = \frac{1}{2} \frac{\sqrt{u}}{1/2} + C = \boxed{\sqrt{x^2-16} + C}$

$u = x^2 - 16$

$du = 2x dx$

38) $\frac{dy}{dx} = x \ln x$

$y = \int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x} dx = \frac{x^2}{2}\ln x - \int \frac{x}{2} dx = \boxed{\frac{x^2}{2}\ln x - \frac{x^2}{4} + C}$

$u = \ln x \quad dv = x$

$du = \frac{1}{x} \quad v = \frac{x^2}{2}$

40) $\frac{dy}{dx} = \frac{-2(x+5)}{(x+2)(x-4)}$

$\frac{-2(x+5)}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4} \Rightarrow -2x-10 = A(x-4) + B(x+2)$

$\Rightarrow -2x-10 = (A+B)x + (2B-4A)$

$-2(A+B=-2) \rightarrow -2A-2\cancel{B}=4$

$-4A+2B=-10 \rightarrow -4A+\cancel{2B}=-10$

$-6A=-6$

$A=1$

$B=-3$

$y = \int \frac{-2(x+5)}{(x+2)(x-4)} dx = \int \left(\frac{1}{x+2} - \frac{3}{x-4}\right) dx$

$= \boxed{\ln|x+2| - 3\ln|x-4|}$

60) $\begin{cases} \frac{dP}{dt} = kP \\ P(0) = P_0 \end{cases}$

a) $P(t) = Ce^{kt}$. Find C.

$P_0 = P(0) = Ce^0 = C$

$\longrightarrow \boxed{P_0 = C}$

$\therefore \boxed{P(t) = P_0 e^{kt}}$

b) $P(8) = 2P_0$. Find k.

$2P_0 = P(8) = P_0 e^{8k}$

$\Rightarrow 2 = e^{8k}$

$\Rightarrow \ln(2) = 8k$

$\Rightarrow \boxed{k = \frac{\ln(2)}{8}}$

64) $u(x,y) = \ln\sqrt{x^2+y^2} = \frac{1}{2}\ln(x^2+y^2)$

$u_x = \frac{1}{2}\frac{1}{x^2+y^2}\cdot 2x = \frac{x}{x^2+y^2}$

$u_{xx} = \frac{(x^2+y^2)\cdot(1) - x(2x)}{(x^2+y^2)^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$

$u_y = \frac{1}{2}\frac{1}{x^2+y^2}\cdot 2y = \frac{y}{x^2+y^2}$

$u_{yy} = \frac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2} = \frac{-y^2+x^2}{(x^2+y^2)^2}$

$u_{xx} + u_{yy} = \frac{-x^2+y^2}{(x^2+y^2)^2} + \frac{-y^2+x^2}{(x^2+y^2)^2} = 0$ ✓