Section 2.2 # 2, 6, 16, 22, 30, 32, 36, 44,

KEY

2) $\frac{1}{2}t^{-1/2}dt + y^2 dy = 0$

$\Rightarrow \frac{1}{2}t^{-1/2}dt = -y^2 dy$

$\Rightarrow \int \frac{1}{2}t^{-1/2}dt = \int -y^2 dy$

$\Rightarrow \frac{1}{2}\frac{t^{1/2}}{1/2} = \frac{-y^3}{3} + C$

$\Rightarrow t^{1/2} = \frac{-y^3}{3} + C$

$\Rightarrow \boxed{t^{1/2} + \frac{y^3}{3} = C}$

6) $(6t^{-9} - 6t^{-3} + t^7)dt + (9 + s^{-2} - 4s^8)ds = 0$

$\Rightarrow \int (6t^{-9} - 6t^{-3} + t^7)\,dt = \int (4s^8 - s^{-2} - 9)\,ds$

$\Rightarrow \frac{6t^{-8}}{-8} - \frac{6t^{-2}}{-2} + \frac{t^8}{8} = \frac{4s^9}{9} - \frac{s^{-1}}{-1} - 9s + C$

$\Rightarrow \boxed{-\frac{6}{8}t^{-8} + 3t^{-2} + \frac{t^8}{8} = \frac{4s^9}{9} + s^{-1} - 9s + C}$

16) $\frac{dy}{dt} = e^{2y+10t}$

$\Rightarrow e^{-2y}dy = e^{10t}dt$

$\Rightarrow \int e^{-2y}dy = \int e^{10t}dt$

$\Rightarrow \frac{e^{-2y}}{-2} = \frac{e^{10t}}{10} + C$

$\Rightarrow C - \frac{e^{-2y}}{2} = \frac{e^{10t}}{10}$

$\Rightarrow 10\left(C - \frac{e^{-2y}}{2}\right) = e^{10t}$

$\Rightarrow \ln(10C - 5e^{-2y}) = 10t$

$\Rightarrow \boxed{t = \frac{\ln(10C - 5e^{-2y})}{10}}$

22) $\frac{dy}{dt} = \frac{t^3}{y\sqrt{(1-y^2)(t^4+9)}}$

$\Rightarrow y\sqrt{1-y^2}\,dy = \frac{t^3}{\sqrt{t^4+9}}dt$

$\Rightarrow \int y\sqrt{1-y^2}\,dy = \int \frac{t^3}{\sqrt{t^4+9}}dt$

$u = 1-y^2$ $\quad w = t^4+9$

$du = -2y\,dy$ $\quad dw = 4t^3 dt$

$\Rightarrow -\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{4}\int \frac{1}{\sqrt{w}}dw$

$\Rightarrow -\frac{1}{2}\frac{u^{3/2}}{3/2} = \frac{1}{4}\frac{w^{1/2}}{1/2} + C$

$\Rightarrow -\frac{1}{3}u^{3/2} = \frac{1}{2}w^{1/2} + C$

$\Rightarrow \boxed{-\frac{1}{3}(1-y^2)^{3/2} = \frac{1}{2}(t^4+9)^{1/2} + C}$

30) $\frac{dy}{dt} = \frac{5^{-t}}{y^2}$

$\Rightarrow y^2 dy = 5^{-t}dt$

$\Rightarrow \int y^2 dy = \int 5^{-t}dt$

$u = -t$

$du = -dt$

$\Rightarrow \frac{y^3}{3} + C = \int -5^u du$

$\Rightarrow \frac{y^3}{3} + C = \frac{-5^u}{\ln 5}$

$\Rightarrow \boxed{\frac{y^3}{3} + C = \frac{-5^{-t}}{\ln 5}}$

32) $dy = (y^2 - 3y + 2)dx$

$\Rightarrow \frac{1}{y^2-3y+2}dy = dx$

$\Rightarrow \int \frac{1}{(y-2)(y-1)}dy = \int dx$

$\frac{1}{(y-2)(y-1)} = \frac{A}{y-2} + \frac{B}{y-1}$

$\Rightarrow 1 = A(y-1) + B(y-2)$

$\Rightarrow 1 = (A+B)y - (A+2B)$

$A + B = 0$

$-A - 2B = 1$

$-B = 1$

$B = -1$

$A = 1$

$\Rightarrow \int\left(\frac{1}{y-2} - \frac{1}{y-1}\right)dy = \int dx$

$\Rightarrow \ln|y-2| - \ln|y-1| = x + C$

$\Rightarrow \boxed{\ln\left|\frac{y-2}{y-1}\right| = x + C}$

36) $\frac{dy}{dt} = y^3 - 1$

$\Rightarrow \frac{1}{y^3-1}dy = dt$

$\Rightarrow \int \frac{1}{(y-1)(y^2+y+1)}dy = \int dt$

$\frac{1}{(y-1)(y^2+y+1)} = \frac{A}{y-1} + \frac{By+C}{y^2+y+1}$

$\Rightarrow 1 = A(y^2+y+1) + (By+C)(y-1)$

$\Rightarrow 1 = (A+B)y^2 + (A-B+C)y + (A-C)$

① $A - C = 1 \rightsquigarrow A = C+1$

② $A - B + C = 0$

③ $A + B = 0 \rightsquigarrow C + 1 + B = 0$

$B = -1 - C$

② $A - B + C = 0$

$\Rightarrow C + 1 + 1 + C + C = 0$

$\Rightarrow 3C + 2 = 0$

$\Rightarrow C = -2/3$

$B = -1 + 2/3 = -1/3$

$A = -2/3 + 1 = 1/3$

$\Rightarrow \int\left(\frac{1}{3}\cdot\frac{1}{y-1} + \frac{-1/3\,y - 2/3}{y^2+y+1}\right)dy = t + C$

$\Rightarrow \frac{1}{3}\int\left(\frac{1}{y-1} - \frac{y+2}{y^2+y+1}\right)dy = t + C$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{3}\int\left(\frac{y+2}{y^2+y+1}\right)dy = t + C$

$u = y^2 + y + 1$

$du = 2y + 1$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{6}\int\left(\frac{2y+1}{y^2+y+1} + \frac{3}{y^2+y+1}\right)dy = t + C$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{6}\ln|y^2+y+1| - \frac{1}{2}\int \frac{1}{y^2+y+1}dy = t + C$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{6}\ln|y^2+y+1| - \frac{1}{2}\int \frac{1}{(y+\frac{1}{2})^2 + \frac{3}{4}}dy = t + C$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{6}\ln|y^2+y+1| - \frac{1}{2(3/4)}\int \frac{1}{\left(\frac{2}{\sqrt{3}}y + \frac{1}{\sqrt{3}}\right)^2 + 1}dy = t + C$

$u = \frac{2}{\sqrt{3}}y + \frac{1}{\sqrt{3}}$ $\quad du = \frac{2}{\sqrt{3}}dy$

$\Rightarrow \frac{1}{3}\ln|y-1| - \frac{1}{6}\ln|y^2+y+1| - \frac{2}{3}\int \frac{1}{u^2+1}\cdot\frac{\sqrt{3}}{2}du = t + C$

$\Rightarrow \boxed{\frac{1}{3}\ln|y-1| - \frac{1}{6}\ln|y^2+y+1| - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}y + \frac{1}{\sqrt{3}}\right) = t + C}$

44) $\begin{cases} \sin^2 y \, dy = dx \\ x(0) = 0. \end{cases}$

$\int \sin^2 y \, dy = \int dx$

$\Rightarrow \int \frac{1-\cos(2y)}{2} dy = \int dx$

$\Rightarrow \frac{1}{2}y - \frac{1}{2}\int \cos(2y) dy = x + C$

$u = 2y$
$du = 2dy$

$\Rightarrow \frac{1}{2}y - \frac{1}{2}\int \cos(u) \cdot \frac{1}{2} du = x + C$

$\Rightarrow \boxed{\frac{1}{2}y - \frac{1}{4}\sin(2y) = x + C}$

$x(0) = 0$:

$\frac{1}{2}(0) - \frac{1}{4}\overset{0}{\cancel{\sin(0)}} - C = x = 0$

$\Rightarrow C = 0$

$\boxed{\frac{1}{2}y - \frac{1}{4}\sin(2y) = x}$