HW#4

1 a) $(2xy+3)dx+(x^2-1)dy=0$

$$\frac{\partial M}{\partial y}=2x \;=\!=\!= \; \frac{\partial N}{\partial x}=2x \quad \checkmark$$

Exact.

$$\frac{\partial F}{\partial x}=2xy+3 \Rightarrow F(x,y)=\int(2xy+3)dx$$

$$\Rightarrow F(x,y)=x^2y+3x+g(y)$$

$$\frac{\partial F}{\partial y}=x^2+g'(y)=x^2-1$$

$$\Rightarrow g'(y)=-1$$

$$\Rightarrow g(y)=-y+C$$

$$F(x,y)=x^2y+3x-y+C$$

$$x^2y+3x-y=C$$

$$\Rightarrow y(x^2-1)=C-3x \Rightarrow \boxed{y=\frac{C-3x}{x^2-1}}$$

b) $(\frac{t}{y})dy=(1+\ln(y))dt=0$

$$\Rightarrow (1+\ln(y))dt+(\frac{t}{y})dy=0$$

$$\frac{\partial M}{\partial y}=\frac{1}{y} \;=\!=\!=\; \frac{\partial N}{\partial t}=\frac{1}{y} \quad \checkmark$$

Exact

$$\frac{\partial F}{\partial y}=\frac{t}{y} \Rightarrow F(x,y)=\int \frac{t}{y}dy$$

$$=t\ln(y)+g(t)$$

$$\frac{\partial F}{\partial t}=\ln(y)+g'(t)=\ln y+1$$

$$\Rightarrow g'(t)=1$$

$$\Rightarrow g(t)=t+C$$

$$F(x,y)=t\ln(y)+t+C$$

$$\Rightarrow t\ln(y)+t=C$$

$$\Rightarrow t(\ln(y)+1)=C$$

$$\Rightarrow \ln(y)+1=\frac{C}{t}$$

$$\Rightarrow \ln(y)=\frac{C}{t}-1$$

$$\Rightarrow \boxed{y=e^{\frac{C}{t}-1}}$$

c) $(\frac{1}{y})dx-(3y-\frac{x}{y^2})dy=0$

$$\Rightarrow (\frac{1}{y})dx+(\frac{x}{y^2}-3y)dy=0$$

$$\frac{\partial M}{\partial y}=-\frac{1}{y^2} \neq \frac{\partial N}{\partial x}=\frac{1}{y^2}$$

Not Exact

d) $(\frac{2}{\sqrt{1-x^2}}+y\cos(xy))dx+(x\cos(xy)-y^{-1/3})dy=0$

$$\frac{\partial M}{\partial y}=\cos(xy)-y\sin(xy)\cdot x$$

$\parallel$ ✓ Exact

$$\frac{\partial N}{\partial x}=\cos(xy)-x\sin(xy)\cdot y$$

$$\frac{\partial F}{\partial x}=\frac{2}{\sqrt{1-x^2}}+y\cos(xy)$$

$$\Rightarrow F(x,y)=\int\left[\frac{2}{\sqrt{1-x^2}}+y\cos(xy)\right]dx$$

$$=2\sin^{-1}(x)+\int y\cos(xy)dx$$

$u=xy$, $du=ydx$

$$=2\sin^{-1}(x)+\int\cos(u)du$$

$$=2\sin^{-1}(x)+\sin(xy)+g(y).$$

$$\frac{\partial F}{\partial y}=\cos(xy)\cdot x+g'(y)$$

$$=x\cos(xy)-y^{-1/3}$$

$$\Rightarrow g'(y)=-y^{-1/3}$$

$$\Rightarrow g(y)=\frac{-3}{2}y^{2/3}+C$$

$$F(x,y)=2\sin^{-1}(x)+\sin(xy)-\frac{3}{2}y^{2/3}+C$$

$$\boxed{2\sin^{-1}(x)+\sin(xy)-\frac{3}{2}y^{2/3}=C}$$

2a) $\begin{cases}(\frac{1}{x}+2y^2x)dx+(2yx^2-\cos y)dy=0\\ y(1)=\pi\end{cases}$

$$\frac{\partial M}{\partial y}=4yx \;=\!=\!=\; \frac{\partial N}{\partial x}=4yx \quad \checkmark$$

Exact

$$\frac{\partial F}{\partial x}=\frac{1}{x}+2y^2x \Rightarrow F(x,y)=\int(\frac{1}{x}+2y^2x)dx$$

$$\Rightarrow F(x,y)=\ln|x|+y^2x^2+g(y)$$

$$\frac{\partial F}{\partial y}=2yx^2+g'(y)=2yx^2-\cos y$$

$$\Rightarrow g'(y)=-\cos y$$

$$\Rightarrow g(y)=-\sin y+C$$

$$F(x,y)=\ln|x|+x^2y^2-\sin(y)+C$$

$$\boxed{\ln|x|+y^2x^2-\sin(y)=C}$$

b) $\begin{cases} (ye^{xy} - \frac{1}{y})dx + (xe^{xy} + \frac{x}{y^2})dy = 0 \\ y(1) = 1 \end{cases}$

$$\frac{\partial M}{\partial y} = e^{xy} + xye^{xy} + \frac{1}{y^2}$$

$$\frac{\partial N}{\partial x} = e^{xy} + xye^{xy} + \frac{1}{y^2}$$

$\parallel$ ✓ Exact

$$\frac{\partial F}{\partial x} = ye^{xy} - \frac{1}{y} \Rightarrow F(x,y) = \int (ye^{xy} - \frac{1}{y})dx$$

$u = xy,\ du = y\,dx$

$$\Rightarrow F(x,y) = \int e^u du - \frac{x}{y}$$

$$\Rightarrow F(x,y) = e^{xy} - \frac{x}{y} + g(y)$$

$$\frac{\partial F}{\partial y} = xe^{xy} + \frac{x}{y^2} + g'(y) = xe^{xy} + \frac{x}{y^2}$$

$\Rightarrow g'(y) = 0$

$\Rightarrow g(y) = c$

$F(x,y) = e^{xy} - \frac{x}{y} + c$

$\boxed{e^{xy} - \frac{x}{y} = c}$

$e^1 - \frac{1}{1} = c$

$e - 1 = c$

$\boxed{e^{xy} - \frac{x}{y} = e - 1}$

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c) $(y^2 \sin x)dx + (\frac{1}{x} - \frac{y}{x})dy = 0$

$y(\pi) = 1$

$$\frac{\partial M}{\partial y} = 2y\sin x \neq \frac{\partial N}{\partial x} = -\frac{1}{x^2} + \frac{y}{x^2}$$

Not Exact

Rewrite: $\frac{dy}{dx} = \frac{y^2 \sin x}{\frac{y}{x} - \frac{1}{x}} = \frac{xy^2 \sin x}{y-1}$

$$\Rightarrow \frac{y-1}{y^2}dy = x\sin x\,dx$$

$$\Rightarrow \int \left(\frac{1}{y} - \frac{1}{y^2}\right)dy = \int x\sin x\,dx$$

$u = x \quad dv = \sin x$

$du = dx \quad v = -\cos x$

$$\Rightarrow \ln|y| + \frac{1}{y} = -x\cos x - \int -\cos x\,dx$$

$$\Rightarrow \ln|y| + \frac{1}{y} = -x\cos x + \sin x + C$$

$$\cancel{\ln|1|}^{0} + \frac{1}{1} = -\pi\cos\pi + \cancel{\sin\pi}^{0} + C$$

$\Rightarrow 1 = \pi + C$

$\Rightarrow 1 - \pi = C$

$$\boxed{\ln|y| + \frac{1}{y} = -x\cos x + \sin x + 1 - \pi}$$