

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

1. Find the Laplace transform of the given functions.

(a) $-20 \mathcal{U}(t - 4)$

(b) $3 \mathcal{U}(t - 5) + 6 \mathcal{U}(t - 8)$

(c) $-12(t + 2)^3 \mathcal{U}(t - 3)$

(d) $-2 \cos(t - \frac{\pi}{4}) \mathcal{U}(t - \frac{\pi}{4})$

(e) $f(t) = \begin{cases} 1, & 0 \leq t < 2 \\ 3 - t, & 2 \leq t < 3 \end{cases}$.
and $f(t) = f(t - 3)$ if $t \geq 3$.

2. Find the inverse Laplace transform of the given functions.

(a) $\frac{2e^{2s} - 5}{se^{5s}}$

(b) $\frac{se^{-3s}}{s^2 + 9}$

(c) $\frac{se^{-3s}}{s^2 + 4s + 5}$

(d) $\frac{e^{-3s}(s - 5)}{(s + 1)(s + 2)}$

3. Solve the given initial value problems using the method of Laplace transforms.

(a) $\begin{cases} y'' + 4y = g(t) \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$.
where $g(t) = \begin{cases} \sin(t), & 0 \leq t \leq 2\pi \\ 0, & 2\pi < t \end{cases}$.

(b) $\begin{cases} y'' + 4y' + 4y = \mathcal{U}(t - \pi) - \mathcal{U}(t - 2\pi) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$.