

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

1. Given that $\mathbf{A} = \begin{pmatrix} 2 & 0 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}$, find $7\mathbf{A} - 4\mathbf{B}$.

2. Given that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 4 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 3 & 1 \end{pmatrix}$, find \mathbf{AB} and \mathbf{BA} .

3. Compute the inverses for the following.

(a) $\mathbf{A} = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix}$

(b) $\mathbf{B} = \begin{pmatrix} e^{3t} & 1 & t \\ 3e^{3t} & 0 & 1 \\ 9e^{3t} & 0 & 0 \end{pmatrix}$

4. Determine the eigenvalues and eigenvectors for:

(a) $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{pmatrix}$

(b) $\mathbf{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

5. Find $\frac{d\mathbf{x}}{dt}$ for $\mathbf{x}(t) = \begin{pmatrix} e^{-t} \sin(3t) \\ 0 \\ -e^{-t} \sin(3t) \end{pmatrix}$.

6. Verify that $\mathbf{x}(t) = \begin{pmatrix} e^{2t} & e^{3t} \\ -e^{2t} & -2e^{3t} \end{pmatrix}$ satisfies $\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \mathbf{x}$.

7. Given that $\mathbf{A} = \begin{pmatrix} 1 & e^{-2t} \\ 3 & e^{-2t} \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} e^{-t} & e^{-t} \\ -e^{-t} & 3e^{-t} \end{pmatrix}$, find:

(a) $\int \mathbf{A}(t) dt$

(b) $\frac{d}{dt}(\mathbf{A}(t)\mathbf{B}(t))$