

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

1. Solve the differential equation:

$$x \sin(x^2) dx - \frac{\cos(\sqrt{y})}{\sqrt{y}} dy = 0$$

$$\int x \sin(x^2) dx = \int \frac{\cos(\sqrt{y})}{\sqrt{y}} dy$$

$$u = x^2 \\ du = 2x dx$$

$$w = \sqrt{y} \\ dw = \frac{1}{2\sqrt{y}} dy$$

$$\Rightarrow \frac{1}{2} \int \sin(u) = 2 \int \cos(w) dw$$

$$\Rightarrow -\frac{1}{2} \cos(u) + C = 2 \sin(w)$$

$$\Rightarrow -\frac{1}{2} \cos(x^2) + C = 2 \sin(\sqrt{y})$$

$$\Rightarrow \frac{-\frac{1}{2} \cos(x^2) + C}{2} = \sin(\sqrt{y})$$

$$\Rightarrow -\frac{1}{4} \cos(x^2) + C_1 = \sin(\sqrt{y})$$

$$\Rightarrow \sin^{-1}\left(-\frac{1}{4} \cos(x^2) + C_1\right) = \sqrt{y}$$

$$\Rightarrow y = \left[\sin^{-1}\left(-\frac{1}{4} \cos(x^2) + C_1\right) \right]^2$$