MATH-266
Quiz #2
06/24/14

Name: Key

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

1. Solve the differential equations:

(a) $(t^2+4)\frac{dy}{dt} + 2ty = 2t \implies \frac{dy}{dt} + \frac{2t}{t^2+4}y = \frac{2t}{t^2+4}$ ← Integrating Factor

$u(t) = e^{\int \frac{2t}{t^2+4}dt} = e^{\int \frac{1}{u}du} = e^{\ln|u|} = u = t^2+4$

$u = t^2+4,\ du = 2t\,dt$

$\implies (t^2+4)\frac{dy}{dt} + 2t\,y = 2t$ By multiplying by (t^2+4) to this eqn

$\frac{d}{dt}\left((t^2+4)y\right) = 2t$

$(t^2+4)y = \int 2t\,dt$

$= t^2 + C$

$\implies \boxed{y = \frac{t^2+C}{t^2+4}}$

(b) $\underbrace{\left(2x + \frac{y}{1+x^2y^2}\right)}_{M}dx + \underbrace{\left(\frac{x}{1+x^2y^2} - 2y\right)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 0 + \frac{(1+x^2y^2)(1) - y(2x^2y)}{(1+x^2y^2)^2} = \frac{1-x^2y^2}{(1+x^2y^2)^2}$))) Exact

$\frac{\partial N}{\partial x} = \frac{(1+x^2y^2)(1) - x(2xy^2)}{(1+x^2y^2)^2} - 0 = \frac{1-x^2y^2}{(1+x^2y^2)^2}$

$\frac{\partial F}{\partial x} = 2x + \frac{y}{1+x^2y^2} \implies F(x,y) = \int \left(2x + \frac{y}{1+(xy)^2}\right)dx$

$= x^2 + \int \frac{y}{1+(xy)^2}dx$

$u = xy,\ du = y\,dx$

$= x^2 + \int \frac{1}{1+u^2}du$

$= x^2 + \tan^{-1}(u) + g(y)$

$= x^2 + \tan^{-1}(xy) + g(y)$

$\frac{\partial F}{\partial y} = 0 + \frac{1}{1+(xy)^2}\cdot x + g'(y) = \frac{x}{1+x^2y^2} - 2y$

$\implies g'(y) = -2y$

$\implies g(y) = -y^2 + C$

$F(x,y) = x^2 + \tan^{-1}(xy) - y^2 + C \implies \boxed{x^2 - y^2 + \tan^{-1}(xy) = C}$