Quiz #5

Solve:

$$y''' + 3y'' - 10y' - 24y = e^{-3t}$$

$$\lambda^3 + 3\lambda^2 - 10\lambda - 24 = 0$$

$$\begin{array}{c|cccc} -2 & 1 & 3 & -10 & -24 \\ & & -2 & -2 & 24 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$$(\lambda+2)(\lambda^2+\lambda-12) = 0$$

$$(\lambda+2)(\lambda+4)(\lambda-3) = 0$$

$$y_h = C_1 e^{-2t} + C_2 e^{-4t} + C_3 e^{3t}$$

$$\left.\begin{aligned} y_p &= Ae^{-3t} \\ y_p' &= -3Ae^{-3t} \\ y_p'' &= 9Ae^{-3t} \\ y_p''' &= -27Ae^{-3t} \end{aligned}\right\} \Rightarrow y_p''' + 3y_p'' - 10y_p' - 24y_p$$

$$= -27Ae^{-3t} + 3(9Ae^{-3t}) - 10(-3Ae^{-3t}) - 24(Ae^{-3t})$$

$$= 6Ae^{-3t}$$

$$= e^{-3t}$$

$$\Rightarrow \boxed{A = 1/6}$$

$$\boxed{y = C_1 e^{-2t} + C_2 e^{-4t} + C_3 e^{3t} + \tfrac{1}{6} e^{-3t}}$$