Quiz #6

Solve: $y''' + y'' = e^t$

$\lambda^3 + \lambda^2 = 0$

$\lambda^2(\lambda+1) = 0$

$$y_h = c_1 + tc_2 + c_3e^{-t}$$

$$W(y_1,y_2,y_3)(t) = \begin{vmatrix} 1 & t & e^{-t} \\ 0 & 1 & -e^{-t} \\ 0 & 0 & e^{-t} \end{vmatrix} = e^{-t}$$

$$W_1(y_1,y_2,y_3) = \begin{vmatrix} t & +e^{-t} \\ 1 & -e^{-t} \end{vmatrix} = -te^{-t} - e^{-t}$$

$$W_2(y_1,y_2,y_3) = \begin{vmatrix} 1 & e^{-t} \\ 0 & -e^{-t} \end{vmatrix} = -e^{-t}$$

$$W_3(y_1,y_2,y_3) = \begin{vmatrix} 1 & t \\ 0 & 1 \end{vmatrix} = 1$$

$$u_1(t) = \int (-1)^{3+1}\left(\frac{-te^{-t} - e^{-t}}{e^{-t}}\right)\cdot e^t\,dt = -\int (t+1)e^t\,dt$$

$u = t+1 \quad | \quad dv = e^t$

$du = dt \quad | \quad v = e^t$

$$= -\left[(t+1)e^t - \int e^t\,dt\right]$$

$$= -(t+1)e^t + e^t$$

$$u_2(t) = \int (-1)^{3+2} \left(\frac{-e^{-t}}{e^{-t}}\right) e^t = \int e^t dt = e^t$$

$$u_3(t) = \int (-1)^{3+3} \left(\frac{1}{e^{-t}}\right) e^t = \int e^{2t} dt$$
$$= \frac{1}{2} e^{2t}$$

$$y = c_1 + tc_2 + c_3 e^{-t} + (e^t - (t+1)e^t) \cdot 1$$
$$+ (e^t)t + \left(\frac{1}{2}e^{2t}\right) \cdot e^{-t}$$

$$y = c_1 + tc_2 + c_3 e^{-t} + e^t - te^t - e^t + e^t t + \frac{1}{2} e^t$$

$$y = c_1 + tc_2 + c_3 e^{-t} + \frac{1}{2} e^t$$