Quiz #7

$$\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}(t)$$

$$\frac{2s^2+10s}{(s^2-2s+5)(s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2+10s = A(s^2-2s+5) + (Bs+C)(s+1)$$

$s=-1$: $-8 = 8A \longrightarrow \boxed{A=-1}$

$s=0$: $0 = 5A + C \Longrightarrow 0 = -5 + C \Longrightarrow \boxed{C=5}$

$s=1$: $12 = 4A + 2(B+C)$

$12 = 4(-1) + 2(B+5)$

$6 = 2B \longrightarrow \boxed{B=3}$

$$\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\}(t) = -\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}(t) + \mathcal{L}^{-1}\left\{\frac{3s+5}{s^2-2s+5}\right\}(t)$$

$$= -e^{-t} + \mathcal{L}^{-1}\left\{\frac{3(s-1)+8}{(s-1)^2+4}\right\}(t)$$

$$= -e^{-t} + 3\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\}(t) + 4\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+4}\right\}(t)$$

$$= \boxed{-e^{-t} + 3e^{t}\cos(2t) + 4e^{t}\sin(2t)}$$