

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Please use equal signs where appropriate and write solutions with a logical flow. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical.

Chapter 4

1. Things you should know:

- Euler's formula, and the basics of complex numbers: addition, multiplication, complex conjugation, modulus and argument.
- The meaning of linear independence/dependence for real-valued functions.
- How to compute a Wronskian, and how to use a Wronskian to detect linear independence/dependence of solutions to linear homogeneous ODE's.
- How to use characteristic polynomials to solve second-order linear homogeneous ODE's with constant coefficients.
- How to solve second-order linear non-homogeneous ODE's using the method of undetermined coefficients.
- How to solve second-order linear non-homogeneous ODE's using the method of variation of parameters.

2. LINEAR HOMOGENEOUS EQUATIONS WITH CONSTANT COEFFICIENTS.

Form: $ay'' + by' + cy = 0$

- Solve the characteristic equation $a\lambda^2 + b\lambda + c = 0$ to find two solutions λ_1 and λ_2 .
- If λ_1, λ_2 are distinct real solutions, then a solution to the ODE is $y = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t}$.
- If $\lambda_1 = \lambda_2$ is a repeated real solutions, then a solution to the ODE is $y = C_1e^{\lambda_1 t} + C_2te^{\lambda_1 t}$.
- If $\lambda_1 = a + bi$ and $\lambda_2 = a - bi$ are complex conjugate solutions, then a solutions to the ODE is $y = C_1e^{at} \cos(bt) + C_2e^{at} \sin(bt)$.

3. LINEAR EQUATIONS: UNDETERMINED COEFFICIENTS

Form: $y'' + p(t)y' + q(t)y = f(t)$

- Obtain a fundamental set of solutions $\{y_1, y_2\}$ to the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.
- Based on what $f(t)$ looks like, try to identify the form of the particular solution y_p to the ODE by multiplying by the appropriate t^s .
- Compute y'_p and y''_p , and plug into the ODE to determine the values of the coefficients.
- A general solution to the ODE is $y = y_p + C_1y_1 + C_2y_2$.

4. LINEAR EQUATIONS: VARIATION OF PARAMETERS

Form: $y'' + p(t)y' + q(t)y = f(t)$

- Obtain a fundamental set of solutions $\{y_1, y_2\}$ to the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.
- Compute the Wronskian $W(y_1, y_2)$.
- Set $u'_1 = -\frac{y_2(t)f(t)}{W(y_1, y_2)(t)}$ and $u'_2 = \frac{y_1(t)f(t)}{W(y_1, y_2)(t)}$.
- Integrate to find u_1 and u_2 . Then, $y_p = u_1y_1 + u_2y_2$ is a particular solution to the ODE.
- A general solution is $y = y_p + C_1y_1 + C_2y_2$.