Calculus Review

1\. a) $f'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2} \cdot \frac{d}{dt}\left(\frac{t}{t^2+4}\right)$

$$= \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2}\left(\frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2}\right)$$

$$= \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{-1/2}\left(\frac{t^2+4-2t^2}{(t^2+4)^2}\right)$$

$$= \boxed{\frac{1}{2}\left(\frac{t^2+4}{t}\right)^{1/2}\left(\frac{-t^2+4}{(t^2+4)^2}\right)}$$

b) $g(y) = \left(\frac{y^2}{y+1}\right)^5$

$$g'(y) = 5\left(\frac{y^2}{y+1}\right)^4 \cdot \frac{d}{dy}\left(\frac{y^2}{y+1}\right)$$

$$= 5\left(\frac{y^2}{y+1}\right)^4 \cdot \left(\frac{(y+1)\cdot 2y - y^2(1)}{(y+1)^2}\right)$$

$$= 5\left(\frac{y^2}{y+1}\right)^4 \cdot \left(\frac{y^2+2y}{(y+1)^2}\right)$$

$$= \boxed{\frac{5y^9(y+2)}{(y+1)^6}}$$

c) $y = \sin(\tan(2x))$

$$\boxed{y' = \cos(\tan(2x)) \cdot \sec^2(2x) \cdot 2}$$

or

$$\boxed{y' = \frac{2\cos(\tan(2x))}{\cos^2(2x)}}$$

d) $y = \sec^2(e^t) + \tan^2(e^t)$

$$y' = 2\sec(e^t) \cdot \sec(e^t)\tan(e^t) \cdot e^t$$
$$+ 2\tan(e^t) \cdot \sec^2(e^t) \cdot e^t$$

$$= \boxed{4e^t \tan(e^t)\sec^2(e^t)}$$

or

$$\boxed{y' = \frac{4e^t \sin(e^t)}{\cos^3(e^t)}}$$

e) $y = [x + (x+\sin^2 x)^3]^4$

$$\boxed{y' = 4[x + (x+\sin^2 x)^3]^3 \cdot [1 + 3(x+\sin^2 x)^2] \cdot (1 + 2\sin x \cdot \cos x)}$$

f) $y = \sqrt{\sin(e^{x^2\cos x})}$

$$\boxed{y' = \frac{1}{2}(\sin(e^{x^2\cos x}))^{-1/2} \cdot \cos(e^{x^2\cos x}) \cdot e^{x^2\cos x} \cdot (2x\cos x - x^2\sin x)}$$

2\. a) $\int \sin(x)\cos(\cos(x))\,dx = -\int \cos(u)\,du$

$u = \cos x$
$du = -\sin x\,dx$

$$= -\sin(u) + C$$
$$= \boxed{-\sin(\cos(x)) + C}$$

b) $\int (x+2)e^{-x^2-4x}\,dx = -\frac{1}{2}\int e^u\,du$

$u = -x^2 - 4x$
$du = (-2x-4)\,dx$
$\Rightarrow du = -2(x+2)\,dx$

$$= -\frac{1}{2}e^u + C$$
$$= \boxed{-\frac{1}{2}e^{-x^2-4x} + C}$$

c) $\int \frac{1}{2x^2+72}\,dx = \frac{1}{2}\int \frac{1}{x^2+36}\,dx$

$x = 6\tan\theta$
$dx = 6\sec^2\theta\,d\theta$

$$= \frac{1}{2}\int \frac{1}{36\tan^2\theta + 36} \cdot 6\sec^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{36\sec^2\theta} \cdot 6\sec^2\theta\,d\theta$$

$\frac{x}{6} = \tan\theta$
$\theta = \tan^{-1}\left(\frac{x}{6}\right)$

$$= \frac{1}{12}\int d\theta$$

$$= \frac{1}{12}\theta + C$$

$$= \boxed{\frac{1}{12}\tan^{-1}\left(\frac{x}{6}\right) + C}$$

d) $\int \frac{1}{5x^2+20}\,dx = \frac{1}{5}\int \frac{1}{x^2+4}\,dx$

$x = 2\tan\theta$
$dx = 2\sec^2\theta\, d\theta$

$= \frac{1}{5}\int \frac{1}{4\tan^2\theta+4}\cdot 2\sec^2\theta\, d\theta$

$= \frac{1}{5}\int \frac{1}{4\sec^2\theta}\cdot 2\sec^2\theta\, d\theta$

$= \frac{1}{10}\int d\theta$

$= \boxed{\frac{1}{10}\tan^{-1}\left(\frac{x}{2}\right)+C}$

e) $\int \frac{1}{1-x^3}\,dx = \int \frac{1}{(1-x)(1+x+x^2)}\,dx$

$\frac{1}{(1-x)(1+x+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x+x^2}$

$\Rightarrow 1 = A(1+x+x^2) + (Bx+C)(1-x)$

$x=1 \Rightarrow 1 = A(1+1+1) + (B+C)(0)$

$\Rightarrow 1 = 3A$

$\Rightarrow A = 1/3$

$x=0 \Rightarrow 1 = A(1+0+0) + (C)(1)$

$\Rightarrow 1 = A+C$

$\Rightarrow 1 = 1/3 + C$

$\Rightarrow C = +2/3$

$\therefore 1 = \frac{1}{3}(1+x+x^2) + (Bx + 2/3)(1-x)$

$\Rightarrow \cancel{1} = \cancel{\tfrac{1}{3}} + \frac{1}{3}x + \frac{1}{3}x^2 - Bx^2 + Bx - \frac{2}{3}x + \cancel{\tfrac{2}{3}}$

$\Rightarrow (\frac{1}{3}-B)x^2 + (\frac{1}{3} - \frac{2}{3} + B)x = 0$

$\frac{1}{3} - B = 0 \Rightarrow B = +1/3$

$= \int\left(\frac{1}{3}\cdot\frac{1}{1-x} + \frac{+\frac{1}{3}x+\frac{2}{3}}{1+x+x^2}\right)dx$

$= \frac{1}{3}\int\left(\frac{1}{1-x} + \frac{x+2}{1+x+x^2}\right)dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{3}\cdot\frac{2}{2}\int \frac{x+2}{1+x+x^2}\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\int \frac{2x+4}{1+x+x^2}\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\int \frac{2x+1}{1+x+x^2}\,dx + \frac{1}{6}\int \frac{3}{1+x+x^2}\,dx$

$u = 1+x+x^2$
$du = (2x+1)\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\int \frac{1}{u}\,du + \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+1-(\frac{1}{2})^2}$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\ln|1+x+x^2| + \frac{1}{2}\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\ln|1+x+x^2| + \frac{4}{3}\cdot\frac{1}{2}\int \frac{1}{(\frac{2}{\sqrt{3}}x+\frac{1}{\sqrt{3}})^2+1}\,dx$

$u = \frac{2}{\sqrt{3}}x + \frac{1}{\sqrt{3}}, \quad du = \frac{2}{\sqrt{3}}\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\ln|1+x+x^2| + \frac{2}{3}\int \frac{1}{u^2+1}\cdot\frac{\sqrt{3}}{2}\,dx$

$= -\frac{1}{3}\ln|1-x| + \frac{1}{6}\ln|1+x+x^2| + \frac{\sqrt{3}}{2}\tan^{-1}(u) + C$

$= \boxed{-\frac{1}{3}\ln|1-x| + \frac{1}{6}\ln|1+x+x^2| + \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{2}{\sqrt{3}}x+\frac{1}{\sqrt{3}}\right)+C}$

f) $\int \frac{1}{x^3-16x}\,dx = \int \frac{1}{x(x+4)(x-4)}\,dx$

$= \int\left(\frac{A}{x} + \frac{B}{x+4} + \frac{C}{x-4}\right)dx$

$\frac{1}{x(x+4)(x-4)} = \frac{A}{x} + \frac{B}{x+4} + \frac{C}{x-4}$

$\Rightarrow 1 = A(x+4)(x-4) + Bx(x-4) + Cx(x+4)$

$x=0 \Rightarrow 1 = A(4)(-4) + 0 + 0$

$\Rightarrow \boxed{A = -1/16}$

$x=-4 \Rightarrow 1 = 0 + B(-4)(-8) + 0$

$\Rightarrow B = 1/32$

$x=4 \Rightarrow 1 = 0 + 0 + C(4)(8)$

$\Rightarrow C = 1/32$

$= -\frac{1}{16}\int \frac{1}{x}\,dx + \frac{1}{32}\int \frac{1}{x+4}\,dx + \frac{1}{32}\int \frac{1}{x-4}\,dx$

$= \boxed{-\frac{1}{16}\ln|x| + \frac{1}{32}\ln|x+4| + \frac{1}{32}\ln|x-4| + C}$

g) $\int \sin x\cdot e^x\,dx = \sin x\cdot e^x - \int e^x\cos x\,dx$

$u = \sin x \quad dv = e^x dx$
$du = \cos x\,dx \quad v = e^x$

$u = \cos x \quad dv = e^x dx$
$du = -\sin x\,dx \quad v = e^x$

$= \sin x\cdot e^x - \left(e^x\cos x - \int -e^x\sin x\,dx\right)$

$\Rightarrow \int \sin x\cdot e^x\,dx = \sin x\cdot e^x - e^x\cos x - \int e^x\sin x\,dx$

$\Rightarrow 2\int e^x\sin x\,dx = e^x\sin x - e^x\cos x + C$

$\Rightarrow \boxed{\int e^x\sin x\,dx = \frac{e^x\sin x - e^x\cos x}{2} + C}$

n) $\int x^2 e^x dx = x^2 e^x - \int 2xe^x dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x dx \qquad v = e^x$

$u = x \qquad dv = e^x$

$du = dx \qquad v = e^x$

$= x^2 e^x - 2\left[xe^x - \int e^x dx\right]$

$= \boxed{x^2 e^x - 2xe^x + 2e^x + C}$