A GEOMETRIC AND HOMOLOGICAL LOOK AT FACTORIZATION.

In the spirit of connections between algebra and geometry, this talk will attempt to give a visual/geometric approach to factorization in integral domains.

We say that the integral domain, R, is atomic if every nonzero nonunit of R can be expressed as a product of irreducible elements (or atoms). In the study of factorization, the field of play is usually the class of domains that are atomic, but there is a richer, more general theory if one allows factorization in a more general setting (and indeed, this is necessary if one wants to study divisibility behavior in domains like $\overline{\mathbb{Z}}$ where there are no irreducibles to work with at all!).

In this talk we will discuss some recent work that highlights some situations where properties more general than atomic are necessary (and even desirable). In particular, we will look at divisibility conditions from a graphical point of view by defining a graph that measures some types of factorization behaviors. Additionally, we will discuss how, in this setting, one can define interesting cochain complexes (and corresponding cohomology groups) that measure some of these behaviors.