MATH 720, Algebra I
Exercises 4
Due Fri 21 Sep

Exercise 1. G is a simple abelian group if and only if $G \cong \mathbb{Z}/p\mathbb{Z}$ for some prime number p.

Exercise 2. (a) Compute $[S_3, S_3]$. Find integers $n_1, \ldots, n_t \geqslant 2$ such that $S_3/[S_3, S_3] \cong \mathbb{Z}/n_1\mathbb{Z} \times \cdots \times \mathbb{Z}/n_t\mathbb{Z}$.
(b) Repeat part (a) for S_4.

Exercise 3. (a) Find a composition series for S_3. Show that S_3 is solvable.
(b) Repeat part (a) for S_4.

Exercise 4. Find an example of a group G with subgroups K, H such that $K \trianglelefteqslant H \trianglelefteqslant G$ and such that K is not a normal subgroup of G.