MATH 720, Algebra I
Exercises 6
Due Fri 05 Oct

Exercise 1. (a) If $G/Z(G)$ is cyclic, then G is abelian.
(b) If $|G| = p^2$ for some prime number p, then G is abelian.

Exercise 2. Assume that $G \neq \{e\}$ is a finite p-group.

(a) Show that G has a normal subgroup of order p.
(b) Show that, for each divisor $d \mid p$, G has a subgroup of order d.

Exercise 3. Assume $H \trianglelefteq G$.

(a) If G is nilpotent then H and G/H are nilpotent.
(b) If H and G/H are nilpotent, must G be nilpotent?

Exercise 4. Let G be a finite group of order n.

(a) Show that G is isomorphic to a subgroup of S_n.
(b) Show that, if G is simple and acts nontrivially on a set of k elements, then $n \mid k!$.

Exercise 5. Show that A_4 has no subgroup of order 6.