MATH 720, Algebra I
Exercises 7
Due Fri 12 Oct

Exercise 1. Let G be a finite p-group. Show that for every divisor d of $|G|$, there is a normal subgroup $H \trianglelefteq G$.

Exercise 2. Let G be a finite group and p a prime number. If $H \trianglelefteq G$ and $|H| = p^k$, then H is contained in each p-Sylow subgroup of G.

Exercise 3. Show that there are no simple groups of order 30.

Exercise 4. Let G be a simple group of order 168. Show that G has exactly 48 elements of order 7.