MONOMIAL IDEALS: HOMEWORK 7

Exercise 1. Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X, Y, Z]$ in three variables. Set

$$I = (Z^4, Y^2Z^3, Y^3, XYZ, XY^2, X^2)R$$

and $f = X$. Compute the irredundant parametric decomposition of fJ and find the complete list of fJ-corner elements.

Exercise 2. Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X, Y]$ in two variables. Let

$$J = (X^6, X^4Y, X^3Y^2, Y^6)R = \mathrm{P}_R(X^2Y^5) \cap \mathrm{P}_R(X^3Y) \cap \mathrm{P}_R(X^5)$$

and $I = (X^5, X^4Y^2, XY^3, Y^4)R$. Compute an irredundant parametric decomposition of $(J :_R I)$ and compute $\mathrm{C}_R((J :_R (f)R))$.

Exercise 3. Prove Proposition III.2.17: Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Let J be a monomial ideal of R. For $k = 1, 2, \ldots$ we have $\operatorname{rad}\left(J^{[k]}\right) = \operatorname{rad}(J)$.

Exercise 4. Prove Lemma III.2.19: Let A be a commutative ring with identity and let R be the polynomial ring $R = A[X_1, \ldots, X_d]$ in d variables. Let $f = \mathbf{X}^{\mathbf{m}}$ and $g = \mathbf{X}^{\mathbf{n}}$ for some $\mathbf{m}, \mathbf{n} \in \mathbb{N}^d$. For $i = 1, \ldots, d$ set $p_i = \max\{m_i, n_i\}$, and set $w = \mathbf{X}^{\mathbf{p}}$. Then $(f)R \cap (g)R = (w)R$.