MATH 721, Algebra II
Exercises 4
Due Fri 08 Feb

Exercise 1. Give an example of a field k, a k-vector space V and a k-subspace $W \subseteq V$ such that $\dim_k(W) = \dim_k(V)$ and $W \neq V$.

Exercise 2. Let V and W be finite dimensional vector spaces over a field k, and show that

$$\dim_k(V) + \dim_k(W) = \dim_k(V \cap W) + \dim_k(V + W).$$

Exercise 3. Let K be a field and let $\{L_\lambda\}_{\lambda \in \Lambda}$ be a set of subfields of K. Show that $\cap_\lambda L_\lambda$ is a subfield of K.

Exercise 4.

(a) Let R be a commutative ring with identity and let $A \subseteq R$ be a subring with $1_A = 1_R \neq 0$. Consider R as an A-module via the multiplication in R. Show that, if R is finitely generated as an R-module, then R is finitely generated as an A-algebra.

(b) Let $k \subseteq K$ be a finite field extension. Show that $k \subseteq K$ is a finitely generated field extension.