MATH 721, Algebra II
Exercise 15
Due Fri 02 May

Exercise 1. Let R be an integral domain with field of fractions K. Show that K is an injective R-module. In particular, it follows that $\mathbb{Q}$ is an injective $\mathbb{Z}$-module.

Exercise 2. Show that, if D is a divisible abelian group and $E \leqslant D$, then D/E is divisible.

Exercise 3. Let $\{G_\lambda\}_{\lambda\in\Lambda}$ be a set of abelian groups. Show that $\oplus_{\lambda\in\Lambda} G_\lambda$ is divisible if and only if each G_λ is divisible. In particular, it follows that $\mathbb{Q}^{(\Lambda)}$ is divisible.

Exercise 4. Let R be a ring with identity, and let I, J be unital R-modules. Show that $I \oplus J$ is injective if and only if I and J are both injective.