Midterm 1 review
Math 165
Spring 2009

§2.2. The Limit of a Function.

Be able to compute limits of functions using graphs.

Be able to to find infinite limits of fractional and logarithmic functions.

§2.3. Calculating Limits Using Limit Laws.

Be able to calculate limits using the limit laws: Assume that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. Let c be a constant and let n be a positive integer.

Sum Law: $\lim_{x\to a}[f(x)+g(x)] = \left[\lim_{x\to a} f(x)\right] + \left[\lim_{x\to a} g(x)\right]$

Difference Law: $\lim_{x\to a}[f(x)-g(x)] = \left[\lim_{x\to a} f(x)\right] - \left[\lim_{x\to a} g(x)\right]$

Constant Multiple Law: $\lim_{x\to a}[cf(x)] = c\left[\lim_{x\to a} f(x)\right]$

Product Law: $\lim_{x\to a}[f(x)g(x)] = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right]$

Quotient Law: $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

Power Law: $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$

Polynomial Law: If f is a polynomial, then $\lim_{x\to a} f(x) = f(a)$

Rational Function Law: If f is a rational function (that is, a quotient of two polynomials) and $f(a)$ is defined, then $\lim_{x\to a} f(x) = f(a)$

Root Law: $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ [If n is even, we need $\lim_{x\to a} f(x) > 0$.]

If $h(x) = f(x)$ when $x \neq a$, then $\lim_{x\to a} h(x) = \lim_{x\to a} f(x)$.

Squeeze Theorem: If $f(x) \leq h(x) \leq g(x)$ when x is near a (except possibly at a), and $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$, then $\lim_{x\to a} h(x) = \lim_{x\to a} f(x) = \lim_{x\to a} g(x)$.

§2.4. The Precise Definition of a Limit.

Be able to use the precise definition of the limit (in terms of ϵ and δ) to prove that certain limits exist.

§2.5. Continuity.

Be able to define the term "continuous". Be able to determine where a given function is continuous. The following functions are continuous everywhere they are defined: polynomials, rational functions, root functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions, and compositions of these functions.

Be able to use the Intermediate Value Theorem to show that a given equation has a solution.

§2.6. Limits at Infinity; Horizontal Asymptotes.

Be able to compute limits of the form $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ using the limit laws with the following facts:

$\lim_{x\to\infty} \tan^{-1} x = \pi/2$ $\qquad$ $\lim_{x\to-\infty} \tan^{-1} x = -\pi/2$

$\lim_{x\to\infty} \frac{1}{x^r} = 0$ when r is a positive rational number

$\lim_{x\to-\infty} \frac{1}{x^r} = 0$ when r is a positive rational number such that x^r is defined for all real numbers x.

Also, be able to determine if $\lim_{x\to\infty} f(x) = \infty$ or $-\infty$.

§2.7. Derivatives and Rates of Change.

Be able to state the definition of the derivative: $f'(a) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

Be able to compute $f'(a)$ using the definition.

Be able to find the slope of the tangent line (and the equation of the tangent line) to the graph of a function at a point. Be able to identify points on the graph where the tangent line has certain properties, for instance, where it is horizontal, where it is perpendicular or parallel to a given line, etc.

Given an equation $y = f(x)$, be able to compute the instantaneous rate of change of y with respect to x.

Given a position function, be able to compute the velocity.

§2.8. The Derivative as a Function.

Be able to compute $f'(x)$ using the definition.

Be able to compute $f''(x)$.

Given a position function, be able to compute the acceleration.

Be able to define the term "differentiable".

Be able to determine where a given function is differentiable.

§3.1. Derivatives of Polynomials and Exponential Functions.

Be able to take derivatives of polynomials, and the function $f(x) = e^x$. Be able to take derivatives of combinations of functions using the following:

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}(f(x))$

Sum Rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$

Difference Rule: $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x))$

§3.2. The Product and Quotient Rules.

Be able to compute derivatives of products and quotients using the rules from 3.1 with the following:

Product Rule: $\frac{d}{dx}[f(x)g(x)] = \left[\frac{d}{dx}(f(x))\right] g(x) + f(x)\left[\frac{d}{dx}(g(x))\right]$

Quotient Rule: $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\left[\frac{d}{dx}(f(x))\right] g(x) - f(x)\left[\frac{d}{dx}(g(x))\right]}{[g(x)]^2}$

§3.3. Derivatives of Trigonometric Functions.

Be able to use the formulas from 3.1–3.2 in combination with the following:

$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\cos x] = -\sin x$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$ $\frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Be able to verify the derivative formulas for $\tan x$, $\cot x$, $\sec x$ and $\csc x$ using the quotient rule.

Be able to use the limit formulas $\displaystyle\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$.

§3.4. The Chain Rule.

Be able to use the formulas from 3.1–3.3 in combination with the following:

Chain Rule: $\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x)$.

Exponential Rule: $\frac{d}{dx}(a^x) = a^x \ln a$.

§3.5. Implicit Differentiation.

Be able to compute derivatives of functions defined implicitly using implicit differentiation. Step 1: Set $y = f(x)$. Step 2: Take the derivative of each side. Step 3: Solve for $y' = f(x)$.

Be able to use implicit differentiation to verify the following formulas:

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}[\csc^{-1} x] = -\frac{1}{x\sqrt{x^2-1}}$$

Be able to use these formulas in combination with the formulas from 3.1–3.4.

§3.6. Derivatives of Logarithmic Functions.

Be able to use implicit differentiation to verify the following formulas:

$$\frac{d}{dx}[\ln x] = \frac{1}{x} \qquad \frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$

Be able to use these formulas in combination with the formulas from 3.1–3.5.

Be able to use logarithmic differentiation to differentiate functions of the form $y = f(x)^{g(x)}$. Step 1: Take the natural logarithm of both sides of the equation. Step 2: Take the derivative of both sides. Step 3: Solve for y'.

For practice, work through the following:
homework assignments;
the even-numbered exercises that correspond to the homework exercises;
your old quizzes;
pp. 167–169 # 1–20, 23–36, 40, 42–44, 47, 51;
pp. 262–263 # 1–42, 44, 46, 49–54, 56–61, 65–81, 83–87, 90, 106–112;
the practice exam

Practice midterm.

1. Compute each limit:

(a) $\lim_{x\to 2}(x^2 - 3e^x)$

(b) $\lim_{x\to 2} \dfrac{x^2-4}{x^2+x-6}$

(c) $\lim_{x\to \infty} \dfrac{\sqrt[3]{8x^6+1}}{3x^2+x}$

(d) $\lim_{x\to 0} x\cot(3x)$

(e) $\lim_{x\to 0} x\sin(1/x)\cos(1/x)$

2. Use the definition of the derivative to find $f'(1)$ when $f(x) = x^3 - x$.

3. Compute the derivative of each function:

(a) $f(x) = x^3 \tan^{-1} x$

(b) $g(x) = \sqrt{1+\sqrt{x}}$

(c) $h(x) = \dfrac{\ln(x^4+1)}{e^{\tan x}}$

(d) $k(x) = x^{\cos x}$

4. Find dy/dx when $x^6 + x^3y^2 + y^6 = 1$.

5. Find the points where the tangent line to the curve $y = x^3 - 2x^2 + 1$ is horizontal.

Bonus. Use the precise definition of the limit to prove the Sum Law: If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a}[f(x)+g(x)] = L + M$.