ABSTRACT

This paper describes a study made on the experimental and theoretical behaviour of laterally loaded piles. Model concrete piles were driven into a uniformly prepared sand bed and were loaded to failure while observing the load deflection behaviour. Both linear and non-linear theoretical analyses were carried out to predict the model pile behaviour using the soil parameters observed by laboratory tests. The comparisons of experimental results and theoretical analyses showed encouraging results for the prediction of ultimate lateral load, but the prediction of lateral deflection was not satisfactory and needs further work.

INTRODUCTION

Pile foundations are commonly used in situations with weak shallow soils or large foundation loads. These piles are designed to carry loads, which are the resultants of vertical dead or live load of the structures, and horizontal loads resulting from wind, water pressure or earth pressure.

There are situations where piles are subjected to substantial horizontal loads. Piles in wharves and jetties carrying the impact forces of mooring ships, pile foundations to the bridge piers to which the accelerating or braking forces applied by heavy vehicles especially in railway bridges, and trestles to overhead travelling cranes are acted upon by horizontal forces. Offshore structures also have to resist horizontal thrusts caused by wind and wave actions. A similar situation exists in transmission tower foundations and tall chimneys where wind exerts large horizontal forces; earth-retaining structures also fall under this category.

One should be mindful that in designing laterally loaded piles there are two criteria that must be satisfied. They relate to the ultimate lateral resistance and allowable deflection at working load of the pile. This study focuses on both lateral deflection and ultimate lateral load of model piles when loaded laterally.

THEORETICAL ANALYSIS

Lateral deformation of piles involves large strains in the soil near the ground surface. The lateral deflection of a laterally loaded pile at relatively low load level will increase approximately linearly with applied loads. However the overall load deflection behaviour is distinctively non-linear. The deflection under lateral load may be estimated with reasonable accuracy using subgrade reaction approach.

Subgrade Reaction Approach

The subgrade reaction approach by Broms (1981) is based on idealisation of the soil as a series of horizontal layers, which provide spring supports. Randolph (1991) also takes up the same argument in his lecture series. This approach is simple compared to equivalent cantilever approach in which piles are approximated to cantilevers.

The lateral deflection is often calculated using a horizontal subgrade reaction coefficient (Subgrade Modulus), $k_h$, defined by Terzaghi (1955), by the equation:

$$k_h = \frac{P}{y}$$

where $k_h$ - Horizontal subgrade reaction coefficient, $P$ - Load transferred from the pile to the surrounding soil per unit depth, $y$ - Lateral deflection of the pile at the point considered.

Linear Analysis: Broms and Francis (1968) proposed the following relation for $k_h$:

$$k_h = 0.65 \left( \frac{E_s}{(1-\nu^2)} \right) \left( \frac{E_p B^4}{E_p I_p} \right)^{1/2}$$

where $E_s$ - Modulus of elasticity of the soil, $B$ - Lateral dimension of the pile, $E_p I_p$ - Flexural rigidity of the pile, $\nu$ - Poisson ratio (for sand $\nu \approx 0.35$).

The variation of modulus of elasticity $E_s$ is assumed to be linear with depth for uniform sand. $E_s$ may be obtained from triaxial test results as,

$$E_s = \frac{(\sigma_1 - \sigma_3)}{\Delta l / l}$$

where $(\sigma_1 - \sigma_3)$ - Deviator stress, $(\Delta l / l)$ - Axial strain

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P.J.Ananthanathan, S.Gajan, T.Kanagalingam and H.N.Seneviratne

Department of Civil Engineering, University of Peradeniya, Sri Lanka.
Considering the smoothening effect of \((1/12)^{th}\) power, the above two equations imply that, \(E_s\) varies linearly with depth and so does \(k_h\).

Therefore, \(k_h\) may be expressed as,

\[
k_h = n_h \cdot z
\]

where \(n_h\) - approximately a constant,
\(z\) - Depth below ground surface.

**Non-Linear Analysis:** For cohesionless soils an empirical equation for \(k_h\) was proposed by Mwindo (1992) as follows:

\[
k_h = 0.052 \gamma^{(-0.48)} k_{h\text{ max}}
\]

\(n_h = k_h / L\)

where, \(L\) is the effective length of the pile. \(k_{h\text{ max}}\) can be determined using soil properties obtained, for e.g. from triaxial tests. The average shear strain \(\gamma\) in the above expression may be estimated using the following experimental relation proposed by Matlock and Reese (1960) as,

\[
\gamma = \frac{(1 + v)\eta}{2.5B}
\]

**Lateral Deflection**

Depending on the dimensionless length \(\eta L< 2\) or \(\eta L>4\), two distinguished relations are given by Broms (1981) as,

**For \(\eta L< 2\):**

\[
y = \frac{18H(1 + 1.33 \frac{e}{L})}{L^2n_h}
\]

**For \(\eta L > 4\):**

\[
y = \frac{2.4H}{n_h^{(3/2)}(E_p I_p)^{2/5}}
\]

where \(\eta\) is defined by the expression,

\[
\eta = \frac{n_h}{(E_p I_p)^{1/5}}
\]

Where \(L\), \(e\) and \(H\) are defined as,

\(L\) - Effective length of the pile,
\(e\) - Eccentricity of the pile head,
\(H\) - Lateral load.

Randolph (1991) proposed the following equation for lateral deflection of the pile at the ground level,

\[
y = \frac{2.43Hn_h}{(E_p I_p)^{2/5}} + \frac{1.62Hn_h^2}{(E_p I_p)^{3/5}}
\]

**Ultimate Lateral Load**

Davis and Poulos (1980) proposed the following equation for the calculation of ultimate lateral load for floating piles. This analysis was based on the limit analysis theory.

\[
H_u = \frac{0.5\gamma' B L^3 k_p}{L + e}
\]

\[k_p = \tan^2\left(45 + \frac{\phi}{2}\right)\]

Where \(H_u\) - Ultimate lateral load,
\(\gamma'\) - Dry unit weight of sand,
\(\phi\) - effective angle of internal friction.

**EXPERIMENTAL PROCEDURE**

The test piles were of cylindrical shape and were cast in concrete to two different diameters 50 mm and 75 mm. The length of the piles ranged from 600 mm to 900 mm. Grade 20 concrete mix with maximum aggregate size of 6 mm was used in casting the piles. For 75 mm diameter piles four reinforcement bars of 6 mm diameter were used at a centre to centre spacing of 30 mm. Shear links of 2 mm mild steel wire were provided at 100 mm spacing. For 50 mm diameter piles one axial reinforcement bar of 6 mm diameter was used. The design criterion adopted is that the pile failure and soil failure occur simultaneously during loading. Pile tips were strengthened with steel shoes to prevent damage during driving. Piles were cured in a water bath for 28 days.

The experimental apparatus consisted of a metal box of dimension 3m x 1.8m x 1.5m; the sand deposition apparatus, pile driving unit and the loading arrangement are as shown in fig 1.

![Fig 1: Configuration diagram of experimental apparatus.](image)
The tank was filled with sand using a hopper device to obtain a uniform sand bed according to the method presented by Walker and Whitaker (1967). After uniform sand bed was prepared, piles were driven into the bed using the pile driving unit. Then the piles were loaded laterally using a pulley arrangement and the lateral deflection of the pile at ground level was monitored. Fig 2 illustrates the positions of the piles in the bed and the spacing between them.

Laboratory tests were carried out to find out the properties of the sand required for the analysis. Sieve analysis for a dry sand sample was carried out to obtain the gradation curve from which representative particle diameters \(d_{60}\) and \(d_{10}\) was found to be 0.2 mm and 0.09 mm respectively; the uniformity coefficient was found to be 2.2. The variation of voids ratio of sand bed with depth was obtained by performing in situ density tests. Voids ratio values, at places outside the zone of influence of the piles, were obtained for the depths of 0, 25, 50, 75 and 100 mm from the top surface of the sand bed. The average voids ratio was found to be 0.73 and the corresponding dry unit weight of the sand was 15 kN/m\(^3\).

The effective angle of internal friction of sand was determined by direct shear tests. Owing to the fact that stress levels were low in the tank bed, three samples were tested at low normal stresses of 25, 50 and 100 kPa. Triaxial tests were also conducted for the determination of internal angle of friction and elastic modulus of soil (\(E_s\)), from which the horizontal subgrade reaction modulus \(k_h\) and the coefficient \(n_h\) were calculated. Three samples were prepared and tested at cell pressures of 25, 50 and 100 kPa. The void ratio of the samples for direct shear and triaxial tests was maintained at about 0.7. The effective angle of internal friction of the sand was found to be \(34^\circ\) from these tests. The results of triaxial tests showed that elastic modulus of soil (\(E_s\)) varies linearly with depth. The \(E_s\) values and its variation with depth as determined from these tests used in the analyses.

RESULTS AND COMPARISON

Lateral Deflection

Experimental Load-deflection behaviour of each pile, which is found to be non-linear, is plotted separately. On the same plot, two theoretical predictions using the method of Broms and Randolph are also plotted for linear and non-linear analysis separately. Figs 3 and 4 show the typical experimentally observed behaviour of a model test pile with theoretical analysis for linear and non-linear cases respectively. Other model test piles also show similar type of behaviour.

From the graphs, it can be clearly seen that the observed experimental load - deflection behaviour is highly non-linear with stiffness decreasing with increasing loads. However, no clear failure load is defined in the graph and the failure loads are estimated approximately as indicated in Fig 4. This is the typical behaviour of short piles, as found by many researches, where soil continues to deform without a clear indication of failure.
The comparison between theoretical and experimental observations indicates that Randolph method gives the best agreement in both linear as well as non-linear case. As can be expected non-linear analysis of Randolph shows better agreement with experimental data than linear analysis of Randolph. Predictions given by Broms method are poor and at least acceptable only in the linear case. In all analyses, both Broms and Randolph predictions deviate considerably from experimental results at high loads.

**Ultimate Lateral Load**

The ultimate lateral load found by the experiments was above the theoretical values given by Davis and Poulos (1980). Table 1 gives the theoretical and experimental values of ultimate lateral load for each model piles together with their dimensions.

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>Length (mm)</th>
<th>Diam. (mm)</th>
<th>Ultimate lateral load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Experimental</td>
</tr>
<tr>
<td>1</td>
<td>660</td>
<td>75</td>
<td>890</td>
</tr>
<tr>
<td>2</td>
<td>770</td>
<td>75</td>
<td>1380</td>
</tr>
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<td>3</td>
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<td>50</td>
<td>320</td>
</tr>
<tr>
<td>4</td>
<td>640</td>
<td>50</td>
<td>511</td>
</tr>
<tr>
<td>5</td>
<td>780</td>
<td>50</td>
<td>810</td>
</tr>
</tbody>
</table>

Table 1. Ultimate Lateral Loads of model test piles.

**Normalised Ultimate Lateral Load**

The above results for ultimate lateral load, together with some experimental results obtained from previous studies by using the same apparatus, are normalised to a non-dimensional form using the Davis and Poulos equation and plotted in Fig 5.

![Graph](image)

**Fig 5:** Normalised form of ultimate lateral load.

The graph shows the variation of \(2H_u/\gamma'k_pL^2B\) with \(L/(L+e)\) for theoretical values and experimental values including previous data of 15 model test piles. The experimental values for \(2H_u/\gamma'k_pL^2B\) are above the theoretical values and hence lie on the safe side. However, there is some scatter in experimental results. This may be due to the problems associated with estimating clear failure load and the method of estimation of failure load needs further review. Since, the ultimate lateral load is well predicted for higher values of \(L/(L+e)\), it can be concluded that the theory is most suitable for the lateral loads with small eccentricities; in other words, it is suitable in situations where the moment on the pile is comparatively small.

**CONCLUSIONS**

The comparison of model pile behaviour with predictions from theoretical analyses was useful in identifying the shortcomings and strengths of the analytical methods.

The analytical predictions by Broms method were found to be conservative but poor; the best agreement in this case was given in the linear analysis. Randolph method fared much better than Broms method and gave best predictions in the non-linear case. Overall predictions were better at low load levels in all analyses and the disagreement increased with load.

The comparison of observed and predicted ultimate load of the piles showed that Davis and Poulos method can be used to obtain a conservative estimate of the failure load for design purposes. The difference between the predicted and observed failure load was not substantial and may be partly due to the difficulties experienced in estimating the failure load. The trend in experimental and predicted results expressed in normalised form indicated that better agreement is obtained when load eccentricity is small.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


