Modeling of Nonlinear Cyclic Load-Deformation Behavior of Shallow Foundations

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Ph.D. Qualifying Exam – March, 2003
Overview

- Centrifuge model tests
- Experimental findings
- Macro-element modeling
- Summary
- Plans for future work
Purpose of the Project

- To study the effects of soil-structure interaction in shallow foundations during dynamic loading

- Explore the nonlinear load-displacement behavior of shallow foundations under combined (V-H-M) loading
  - Vertical force – settlement
  - Horizontal force – sliding
  - Moment – rotation

- Provide data base containing all experimental data

- Develop numerical models that can be used in design
Container and Test Setup

Station A  (Vertical Load Test)

Station B  (Horizontal Load, Standard Ht)

Station C

LP

Load Cell

ACTUATOR

Model Shear Wall

Load Cell

Transverse Teflon Supports

beams continue for length of box

Nevada Sand

200
Model Variables

- Wall Weight
- Footing Area
- Single Wall / Double Wall
- Embedment
- Horizontal Push Height
- Soil Properties
- Loading type
Loading Types

• Vertical Slow Cyclic

• Horizontal Slow Cyclic

• Dynamic Loading
Vertical Slow Cyclic

Actuator / Load Cell

Single Footing

Vertical Linear Potentiometers

Double Footing
Horizontal Slow Cyclic

Actuator / Load Cell

Linear Potentiometers

Teflon® Supports
Dynamic

Double Wall

Single Wall

Accelerometer
Table 3.4: Test Event Summary

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Soil Type</th>
<th>Soil Strength ($\phi$ or $c_u$)</th>
<th>Event Type$^2$</th>
<th>Static $FS_v$</th>
<th>Footing Length (m)</th>
<th>Footing Width (m)</th>
<th>Embed Depth [m]</th>
<th>Load Height [m]</th>
<th>Wall Type</th>
<th>Oil or No?</th>
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<td>Dry Sand (Dr = 60%~80%)</td>
<td>$\phi = 37.7^\circ$</td>
<td>VSC 3.8</td>
<td>2.672</td>
<td>0.686</td>
<td>0.3</td>
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1See section 3.1.3 for determination of $\phi$ and $c_u$

2HSC: Horizontal Slow Cyclic
VSC: Vertical Slow Cyclic
Presentation of Results

s: vertical displacement (settlement)
u: horizontal displacement (sliding)
θ: rotation
V: vertical load
H: horizontal load
M: moment
Data from test SSG02, test#3a, FS = 6.7, embedment = 0.0m, load height = 4.9m, footing length = 2.84m
Data from test SSG03, test#8c, d, and e, FS = 6.7, embedment = 0.7m, footing length = 2.84m
Comparison of dynamic data trend line with slow-cyclic moment-rotation. KRRO02, test KRR02_S21 after Rosebrook, 2001 (Left) and SSG02, test#3a (Right).
Macro-Element Modeling

- Considers foundation and soil as a single macro-element
- Constitutive model, based on plasticity theory, relates the forces ($V, H, M$) and displacements ($s, u, \theta$) acting at the center of the footing
Interaction Diagrams

Georgiadis and Butterfield, 1988

- Inclined loading (no eccentricity, $M = 0$)
- Parabolic failure envelopes in $(H - V)$ plane for $M = 0$

- Eccentric loading (no inclination, $H = 0$)
- Parabolic failure envelopes in $(M/B - V)$ plane for $H = 0$
(M-H-V) Failure Envelope

- Combined effect of inclination (H) and eccentricity (M)
- 3-D failure envelope in (M/B - H- V) space
3D Failure Surface

Gottardi and Butterfield, 1993

- Complete 3-D failure envelope
- Elliptical cross sections in constant V planes
- Parabolic sections in (H - V) and (M/B – V) planes
Macro-Element Plasticity Models

- Nova and Montrasio (1991)
- Houlsby and Cassidy (2002)
- Cremer et. al. (2001)
Cremer. et. al. (2001)

- A macro-element model:
  - Cyclic loading
  - Cohesive soils
  - Calibrated using Dynaflow numerical simulations

- Properties:
  - Material nonlinearities – soil yielding (plasticity model)
  - Geometrical nonlinearities – footing uplift (uplift model)
  - Coupling between models account for the influence of yielding on uplift and vice versa
Global Model

\[
\frac{dF}{d\vec{u}_{\text{tot}}} = dF_{\text{el}} + dF_{\text{pl}} + dF_{\text{up}}
\]

\[
dF = K_{\text{el}plup} d\vec{u}_{\text{tot}}
\]

**Elasticity**

\[
dF, d\vec{u}_{\text{el}}
\]

**Plasticity**

\[
dF, d\vec{u}_{\text{pl}}
\]

**Uplift**

\[
dF, d\vec{u}_{\text{up}}
\]

**Plasticity model**

\[
dF = K_{\text{el}pl} (d\vec{u}_{\text{el}} + d\vec{u}_{\text{pl}})
\]

**Uplift model**

\[
dF = K_{\text{up}} d\vec{u}_{\text{up}}
\]
Projections of the failure surface

**Figure 1:** Projections of the failure surface for different models. The graphs depict the relationship between $F_V = (1/FS_V)$ and $F_H$ (top) and $F_M$ (bottom), with data points for SSG02, SSG03, KRR, and the model of Cremer et al. (2001).
Cross sections of failure surface

- SSG02 (FS = 6.7)
- SSG02 (FS = 6.7)
- Cremer. et. al., (2001) (FS = 6.7)
- Cremer. et. al., (2001) (FS = 2.8)
- KRR03 (FS = 2.8)
- KRR02 (FS = 2.8)
Plasticity Model

\[ f = \left( \frac{F_H - \alpha \Gamma_H}{\rho \Gamma_H^\gamma} \right)^2 + \left( \frac{F_M - \beta \Gamma_M}{\rho \Gamma_M^\gamma} \right)^2 - 1 = 0 \]
Yield Surface
Uplift Model

Displacements due to uplift are function of:
- ratio of separation, $\delta$
- amount of rotation when uplift is initiated, $\theta_o$

\[ U_M^{up} = (1 - F_V)\theta_0 \left( \frac{\delta / \delta_{\text{max}}}{1 - \delta / \delta_{\text{max}}} \right)^2 \]

\[ U_V^{up} = (1 - F_V) \frac{\theta_0}{2} \left( \frac{\delta / \delta_{\text{max}}}{1 - \delta / \delta_{\text{max}}} + \ln\left(1 - \delta / \delta_{\text{max}}\right) \right) \]
Uplift Surfaces

- $M'_0 (\delta=0\%)$
- $M'_c (\delta=100\%)$
- Failure surface
- 'Iso'-uplift surface

$M'_{\text{max}}$

$\delta_{\text{max}}$

$\frac{dM'}{dV'}=1/2$

$\frac{dM'}{dV'}=1/4$
Uplift Model

The diagram illustrates the Uplift Model with various surfaces and failure criteria. The uplift surface, loading surface, and failure criterion are marked with corresponding labels. The graphs show the relationship between moment (M) and rotation (δ) for both elastic and plastic regimes. The critical moment (M_c) and maximum moment (M_max) are indicated, along with initial and final rotation points (δ_0⁽ⁱ⁾ and δ_max). The graph also includes an inset showing the moment (M) vs. rotation (θ) for different material properties.
Implementation

- Model is implemented in C++

- Implicit method (backward Euler) analysis with modified Newton-Raphson iteration procedures

- Displacement controlled analysis is complicated
  - \( U_{\text{tot}} = (U_{\text{el}} + U_{\text{pl}}) + (U_{\text{up}}) \)
  - Coupled yielding and uplift
Summary

• Nonlinear cyclic load – displacement behavior of shallow footings in (V-H-M) space is explored using centrifuge model tests

• Structure dimensions, soil properties and loading paths are varied to obtain the footing behavior for different factor of safeties and different loading conditions

• Experimental failure points compare well with the failure envelopes proposed by other researchers

• Macro modeling is consistent with interaction diagrams based on experimental findings and bearing capacity equations
Plans for future work

• One more series of model tests with varying embedment and different loading paths

• Comparing Cremer’s macro-element model predictions with experimental results on sand

• Improving the model (plasticity model components) to capture the footing behavior in (V – H – M) space loading

• An improved Macro-element model may be derived by combining the features Cremer's model and the Houlsby and Cassidy model.