FACTORIAL ARRANGEMENTS

**Factor** - refers to a kind of treatment.
Factors will be referred to with capital letters.

**Level** - refers to several treatments within any factor.
Levels will be referred to with lower case letters.

- A combination of lower case letters and subscript numbers will be used to designate individual treatments (a₀, a₁, b₀, b₁, a₀b₀, a₀b₁, etc.)

- Experiments and examples discussed so far in this class have been single factor experiments.

- For single factor experiments, results obtained are applicable only to the particular level in which the other factor(s) was maintained.

  **Example:** Five seeding rates and one cultivar.

A factorial is not a design but an arrangement.

- A factorial is a study with two or more factors in combination.

- Each level of a factor **must** appear in combination with all levels of the other factors.

- Factorial arrangements allow us to study the **interaction** between two or more factors.

**Interaction:**

1. the failure for the response of treatments of a factor to be the same for each level of another factor.

2. When the **simple effects** of a factor differ by more than can be attributed to chance, the differential response is called an interaction.
Examples of Interactions

No interaction (similar response)  
Interaction (diverging response)

Interaction (crossover response)  
Interaction (converging response)
Simple Effects, Main Effects, and Interactions

- Simple effects, main effects, and interactions will be explained using the following data set:

Table 1. Effect of two N rates of fertilizer on grain yield (Mg/ha) of two barley cultivars.

<table>
<thead>
<tr>
<th>Cultivar (A)</th>
<th>Fertilizer Rate (B)</th>
<th>0 kg N/ha (b₀)</th>
<th>60 kg N/ha (b₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larker (a₀)</td>
<td>1.0 (a₀b₀)</td>
<td>3.0 (a₀b₁)</td>
<td></td>
</tr>
<tr>
<td>Morex (a₁)</td>
<td>2.0 (a₁b₀)</td>
<td>4.0 (a₁b₁)</td>
<td></td>
</tr>
</tbody>
</table>

- The **simple effect** of a factor is the difference between its two levels at a given level of the other factor.

  Simple effect of A at b₀ = a₁b₀ - a₀b₀
  
  \[= 2 - 1\]
  
  \[= 1\]

  Simple effect of A at b₁ = a₁b₁ - a₀b₁
  
  \[= 4 - 3\]
  
  \[= 1\]

  Simple effect of B at a₀ = a₀b₁ - a₀b₀
  
  \[= 3 - 1\]
  
  \[= 2\]

  Simple effect of B at a₁ = a₁b₁ - a₁b₀
  
  \[= 4 - 2\]
  
  \[= 2\]
• The **main effect** of a factor is the average of the simple effects of that factor over all levels of the other factor.

\[
\text{Main effect of } A = \frac{\text{simple effect of } A \text{ at } b_0 + \text{simple effect of } A \text{ at } b_1}{2}
\]
\[
= \frac{(1 + 1)}{2}
\]
\[
= 1
\]

\[
\text{Main effect of } B = \frac{\text{simple effect of } B \text{ at } a_0 + \text{simple effect of } B \text{ at } a_1}{2}
\]
\[
= \frac{(2 + 2)}{2}
\]
\[
= 2
\]

• The **interaction** is a function of the difference between the simple effects of A at the two levels of B divided by two, or vice-versa.

*This works only for 2 x 2 factorials*

\[
A \times B = \frac{1}{2}(\text{Simple effect of } A \text{ at } b_1 - \text{Simple effect of } A \text{ at } b_0)
\]
\[
= \frac{1}{2}(1 - 1)
\]
\[
= 0
\]

or

\[
A \times B = \frac{1}{2}(\text{Simple effect of } B \text{ at } a_1 - \text{Simple effect of } B \text{ at } a_0)
\]
\[
= \frac{1}{2}(2 - 2)
\]
\[
= 0
\]

**Example with an interaction:**

Table 2. Effect of two N rates of fertilizer on grain yield (Mg/ha) of two barley cultivars.

<table>
<thead>
<tr>
<th>Cultivar (A)</th>
<th>Fertilizer Rate (B)</th>
<th>0 kg N/ha (b₀)</th>
<th>60 kg N/ha (b₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larker (a₀)</td>
<td>1.0 (a₀b₀)</td>
<td>1.0 (a₀b₁)</td>
<td></td>
</tr>
<tr>
<td>Morex (a₁)</td>
<td>2.0 (a₁b₀)</td>
<td>4.0 (a₁b₁)</td>
<td></td>
</tr>
</tbody>
</table>

• The **simple effect** of a factor is the difference between its two levels at a given level of the other factor.

\[
\text{Simple effect of } A \text{ at } b_0 = a_1b_0 - a_0b_0
\]
\[
= 2 - 1
\]
\[
= 1
\]
Simple effect of $A$ at $b_1 = a_1b_1 - a_0b_1$
$= 4 - 1$
$= 3$

Simple effect of $B$ at $a_0 = a_0b_1 - a_0b_0$
$= 1 - 1$
$= 0$

Simple effect of $B$ at $a_1 = a_1b_1 - a_1b_0$
$= 4 - 2$
$= 2$

- The **main effect** of a factor is the average of the simple effects of that factor over all levels of the other factor.

Main effect of $A = \frac{(\text{simple effect of } A \text{ at } b_0 + \text{simple effect of } A \text{ at } b_1)}{2}$
$= \frac{(1 + 3)}{2}$
$= 2$

Main effect of $B = \frac{(\text{simple effect of } B \text{ at } a_0 + \text{simple effect of } B \text{ at } a_1)}{2}$
$= \frac{(0 + 2)}{2}$
$= 1$
• The **interaction** is a function of the difference between the simple effects of A at the two levels of B divided by two, or vice-versa.

  *(This works only for 2 x 2 factorials)*

\[
A \times B = \frac{1}{2}(\text{Simple effect of } A \text{ at } b_1 - \text{Simple effect of } A \text{ at } b_0)
\]
\[
= \frac{1}{2}(3 - 1)
\]
\[
= 1
\]

or

\[
A \times B = \frac{1}{2}(\text{Simple effect of } B \text{ at } a_1 - \text{Simple effect of } B \text{ at } a_0)
\]
\[
= \frac{1}{2}(2 - 0)
\]
\[
= 1
\]

**Facts to Remember about Interactions**

1. An interaction between two factors can be measured **only** if the two factors are tested together in the same experiment.

2. When an interaction is absent, the simple effect of a factor is the same for all levels of the other factors and equals the main effect.

3. When interactions are present, the simple effect of a factor changes as the level of the other factor changes. Therefore, the main effect is different from the simple effects.

**Example of ANOVA for a 2x2 Factorial**

**Table 1.** Data for the RCBD analysis of a 2 x 2 factorial arrangement.

<table>
<thead>
<tr>
<th>Replicate</th>
<th>a₀b₀</th>
<th>a₀b₁</th>
<th>a₁b₀</th>
<th>a₁b₁</th>
<th>Y_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>19</td>
<td>29</td>
<td>32</td>
<td>92</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>22</td>
<td>27</td>
<td>35</td>
<td>99</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>23</td>
<td>33</td>
<td>38</td>
<td>108</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>21</td>
<td>30</td>
<td>37</td>
<td>101</td>
</tr>
<tr>
<td>Y_i</td>
<td>54</td>
<td>85</td>
<td>119</td>
<td>142</td>
<td>400=Y..</td>
</tr>
</tbody>
</table>
Step 1. Calculate Correction Factor

\[ CF = \frac{Y^2}{rab} \]

\[ = \frac{400^2}{4 \times 2 \times 2} \]

\[ = 10,000 \]

Step 2. Calculate Total SS

\[ \text{Total SS} = \sum Y_{ij}^2 - CF \]

\[ = (12^2 + 15^2 + 14^2 + \ldots + 37^2) - CF \]

\[ = 1,170.0 \]

Step 3. Calculate Replicate SS

\[ \text{Rep SS} = \sum \frac{Y_{ij}^2}{ab} - CF \]

\[ = \frac{(92^2 + 99^2 + 108^2 + 101^2)}{(2 \times 2)} - CF \]

\[ = 32.5 \]

Step 4. Partition Treatment SS

Step 4.1. Make Table of Treatment Totals

<table>
<thead>
<tr>
<th></th>
<th>a₀</th>
<th>a₁</th>
<th>ΣB</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₀</td>
<td>54</td>
<td>119</td>
<td>173</td>
</tr>
<tr>
<td>b₁</td>
<td>85</td>
<td>142</td>
<td>227</td>
</tr>
<tr>
<td>ΣA</td>
<td>139</td>
<td>261</td>
<td>400</td>
</tr>
</tbody>
</table>
Step 4.2. Calculate A SS

\[ A\ SS = \sum_{rb} A^2 - CF \]

\[ = \frac{(139^2 + 261^2)}{4 \times 2} - CF \]

\[ = 930.25 \]

Step 4.3. Calculate B SS

\[ B\ SS = \sum_{ra} B^2 - CF \]

\[ = \frac{(173^2 + 227^2)}{4 \times 2} - CF \]

\[ = 182.25 \]

Step 4.4. Calculate A x B SS

\[ AxB\ SS = \sum_{r} ab^2 - CF - A\ SS - B\ SS \]

\[ = \frac{(54^2 + 85^2 + 119^2 + 142^2)}{4} - CF - A\ SS - B\ SS \]

\[ = 4.0 \]

**NOTE:** A SS + B SS + A x B SS = Treatment SS

Step 5. Calculate Error SS

\[ Error\ SS = Total\ SS - Rep\ SS - A\ SS - B\ SS - A\ x\ B\ SS \]

\[ = 21.0 \]
Step 6. Do the ANOVA

<table>
<thead>
<tr>
<th>SOV</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F (assuming A and B fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rep</td>
<td>r - 1 = 3</td>
<td>32.5</td>
<td>10.833</td>
<td>Rep MS/Error MS = 4.64*</td>
</tr>
<tr>
<td>A</td>
<td>a - 1 = 1</td>
<td>930.25</td>
<td>930.25</td>
<td>A MS/Error MS = 398.679**</td>
</tr>
<tr>
<td>B</td>
<td>b - 1 = 1</td>
<td>182.25</td>
<td>182.25</td>
<td>B MS/Error MS = 78.107**</td>
</tr>
<tr>
<td>A x B</td>
<td>(a - 1)(b - 1) = 1</td>
<td>4.00</td>
<td>4.00</td>
<td>AxB MS/Error MS = 1.714</td>
</tr>
<tr>
<td>Error</td>
<td>(r - 1)(ab - 1) = 9</td>
<td>21.00</td>
<td>2.333</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>rab - 1 = 15</td>
<td>1170.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 7. Calculate LDS’s (0.05)

Step 7.1 Calculate LSD_A

\[ \text{LSD}_A = t_{0.05/2;9} \sqrt{\frac{2 \text{Error MS}}{rb}} \]

\[ = 2.262 \sqrt{\frac{2(2.333)}{4 \times 2}} \]

\[ = 1.7 \]

Mean of treatment A averaged across all levels of B.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_0</td>
<td>17.4 a</td>
</tr>
<tr>
<td>a_1</td>
<td>32.6 b</td>
</tr>
</tbody>
</table>

Step 7.2 Calculate LSD_B

\[ \text{LSD}_B = t_{0.05/2;9} \sqrt{\frac{2 \text{Error MS}}{ra}} \]

\[ = 2.262 \sqrt{\frac{2(2.333)}{4 \times 2}} \]

\[ = 1.7 \]

Mean of treatment B averaged across all levels of A.
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₀</td>
<td>21.6 a</td>
</tr>
<tr>
<td>b₁</td>
<td>28.4 b</td>
</tr>
</tbody>
</table>

Step 7.3  
Calculate LSDₐₓₜₜ

\[
\text{LSD } AxB = t_{0.05/2;9df} \sqrt{\frac{\text{Error MS}}{r}}
\]

\[
= 2.262 \sqrt{\frac{2(2.333)}{4}}
\]

\[
= 2.4
\]

- You can see from the figure above that the two lines are nearly parallel. This indicates that B is responding similarly at all levels of A; thus, there is no interaction.
**Example of a CRD with a 4x3 Factorial Arrangement**

Given there are 3 replicates, the SOV and df would be as follows:

<table>
<thead>
<tr>
<th>SOV</th>
<th>Df</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>a-1 = 3</td>
</tr>
<tr>
<td>B</td>
<td>b-1 = 2</td>
</tr>
<tr>
<td>AxB</td>
<td>(a-1)(b-1) = 6</td>
</tr>
<tr>
<td>Error</td>
<td>By subtraction = 24</td>
</tr>
<tr>
<td>Total</td>
<td>rab-1 = 35</td>
</tr>
</tbody>
</table>

**Example of a RCBD with a 4x3x2 Arrangement**

Given there are 5 replicates, the ANOVA would look as follows:

<table>
<thead>
<tr>
<th>SOV</th>
<th>Df</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rep</td>
<td>r-1 = 4</td>
</tr>
<tr>
<td>A</td>
<td>a-1 = 3</td>
</tr>
<tr>
<td>B</td>
<td>b-1 = 2</td>
</tr>
<tr>
<td>C</td>
<td>c-1 = 1</td>
</tr>
<tr>
<td>AxB</td>
<td>(a-1)(b-1) = 6</td>
</tr>
<tr>
<td>AxC</td>
<td>(a-1)(c-1) = 3</td>
</tr>
<tr>
<td>BxC</td>
<td>(b-1)(c-1) = 2</td>
</tr>
<tr>
<td>AxBxC</td>
<td>(a-1)(b-1)(c-1) = 6</td>
</tr>
<tr>
<td>Error</td>
<td>(r-1)(abc-1) = 92</td>
</tr>
<tr>
<td>Total</td>
<td>rabc-1 = 119</td>
</tr>
</tbody>
</table>

- In order to calculate the Sums of Squares for A, B, C, AxB, Ax C, BxC, and AxBxC, you will need to make several tables of treatment totals.

- The general outline of these tables is as follows:

Table 1. Totals used to calculate A SS, B SS, and AxB SS.

<table>
<thead>
<tr>
<th>b_0</th>
<th>a_0</th>
<th>a_1</th>
<th>a_2</th>
<th>a_3</th>
<th>( \sum B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum A )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Remember AxB SS = \( \sum \frac{(ab)^2}{rc} \) - CF – A SS – B SS
Table 2. Totals used to calculate A SS, C SS, and AxC SS.

\[
\begin{array}{cccc}
  a_0 & a_1 & a_2 & a_3 \\
  c_0 & a_0 c_0 & a_1 c_0 & \\
  c_1 & & & \\
\end{array}
\]

\[\sum A\]

Remember \( \text{AxC SS} = \sum (ac)^2 \cdot \frac{rb}{\text{CF}} - \text{A SS} - \text{C SS} \)

Table 3. Totals used to calculate B SS, C SS, and BxC SS.

\[
\begin{array}{ccc}
  b_0 & b_1 & b_2 \\
  c_0 & b_0 c_0 & b_1 c_0 \\
  c_1 & & \\
\end{array}
\]

\[\sum B\]

Remember \( \text{BxC SS} = \sum (bc)^2 \cdot \frac{ra}{\text{CF}} - \text{B SS} - \text{C SS} \)

Table 4. Values used to calculate Total SS, Rep SS, and AxBxC SS.

\[
\begin{array}{ccc}
  \text{Rep 1} & \text{Rep 2} & \text{Rep 3} \\
  a_0 b_0 c_0 & a_0 b_0 c_1 & \\
  a_0 b_1 c_0 & \\
  a_3 b_1 c_1 & \\
\end{array}
\]

\[\sum \text{Rep}\]

Remember \( \text{AxBxC SS} = \sum (abc)^2 \cdot \frac{r}{\text{CF}} - \text{A SS} - \text{B SS} - \text{C SS} - \text{AxB SS} - \text{AxC SS} - \text{BxC SS} \)

**Linear Model**

\[Y_{ijk} = \mu + \nu_i + \alpha_j + \beta_k + \alpha \beta_{jk} + \epsilon_{ijk}\]

Where:
- \(\mu\) = Experiment mean
- \(\nu_i\) = Rep effect if the \(i^\text{th}\) replicate
- \(\alpha_j\) = Effect of the \(j^\text{th}\) level of factor A
- \(\beta_k\) = Effect of the \(k^\text{th}\) level of factor B
- \(\alpha \beta_{jk}\) = A x B interaction effect
- \(\epsilon_{ijk}\) = Random error
Advantages of Factorial Arrangements

1. Provides estimates of interactions.
2. Possible increase in precision due to so-called “hidden replication.”
3. Experimental rates can be applied over a wider range of conditions.

Disadvantages of Factorial Arrangements

1. Some treatment combinations may be of little interest.
2. Experimental error may become large with a large number of treatments.
3. Interpretation may be difficult (especially for 3-way or more interactions).

Randomizing Factorial Arrangements

1. Assign numbers to treatment combinations.
2. Randomize treatments according to design.

Example - RCBD with a 2x4 Factorial Arrangement

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Treatment number</th>
<th>Treatment</th>
<th>Treatment number</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀b₀</td>
<td>1</td>
<td>a₁b₀</td>
<td>5</td>
</tr>
<tr>
<td>a₀b₁</td>
<td>2</td>
<td>a₁b₁</td>
<td>6</td>
</tr>
<tr>
<td>a₀b₂</td>
<td>3</td>
<td>a₁b₂</td>
<td>7</td>
</tr>
<tr>
<td>a₀b₃</td>
<td>4</td>
<td>a₁b₃</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rep 1</th>
<th>3</th>
<th>7</th>
<th>2</th>
<th>6</th>
<th>4</th>
<th>5</th>
<th>1</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a₀b₂</td>
<td>a₁b₂</td>
<td>a₀b₁</td>
<td>a₁b₁</td>
<td>a₀b₃</td>
<td>a₁b₀</td>
<td>a₀b₀</td>
<td>a₁b₃</td>
</tr>
</tbody>
</table>

Interpreting Results of ANOVA Involving Interaction Terms

- Interpretation should always begin with the higher level interaction terms (e.g. three-way interactions before two-interactions, etc.).

- Interpretation of the main effects should never be done before interpreting the interaction terms.
The $F$-test for interaction terms can be significant because of two reasons.

1. True interaction.
2. Differences in magnitude between treatment means.

- Differences in magnitude when responses are similar, but the differences between means are great.

- An example of this would be yield of a series of varieties having the same relative rank at two locations that differ greatly in yield due to differences in growing conditions (e.g. favorable growing conditions vs. drought conditions).

Examples of Interactions and Differences in Magnitude

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**Differences in magnitude**

**Interaction**

---

**Interaction**
Flow Chart for Interpreting ANOVA’s with Interaction Terms

Non-significant interaction

Mention that the interaction was NS and discuss the main effects.

Significant interaction

Determine why the interaction was significant.

Significant due to a “true” interaction.

Discuss the interaction. Discussion of the main effects should be avoided.

Significant due to differences in magnitude.

State that the interaction is significant due to differences in magnitude and discuss the main effects. Discussion of the interaction should be avoided.
SAS Commands for Analyzing an RCBD with a Factorial Arrangement

- Please note that SAS will not calculate LSD values or do mean separation for interactions.

- SAS will only calculate LSD values and do mean separation on the main effects.

- The analysis below assumes factors A, B, and C are all considered fixed effects.

```sas
options pageno=1;

data fact;
  input A $ B $ C$ Trt Rep Yield;
  datalines;
   a0 b0 c0  1  1  12
   a0 b0 c1  2  1  18
   a0 b1 c0  3  1  16
   a0 b1 c1  4  1  24
   a1 b0 c0  5  1  18
   a1 b0 c1  6  1  27
   a1 b1 c0  7  1  23
   a1 b1 c1  8  1  31
   a2 b0 c0  9  1  25
   a2 b0 c1 10  1  36
   a2 b1 c0 11  1  28
   a2 b1 c1 12  1  41
   a0 b0 c0  1  2  22
   a0 b0 c1  2  2  29
   a0 b1 c0  3  2  25
   a0 b1 c1  4  2  37
   a1 b0 c0  5  2  32
   a1 b0 c1  6  2  41
   a1 b1 c0  7  2  32
   a1 b1 c1  8  2  41
   a2 b0 c0  9  2  35
   a2 b1 c0 10  2  51
   a2 b1 c1 11  2  36
   a2 b1 c1 12  2  50
   a0 b0 c0  1  3  25
   a0 b0 c1  2  3  29
   a0 b1 c0  3  3  28
   a0 b1 c1  4  3  37
   a1 b0 c0  5  3  31
   a1 b0 c1  6  3  39
   a1 b1 c0  7  3  35
   a1 b1 c1  8  3  42
   a2 b0 c0  9  3  41
   a2 b0 c1 10  3  49
   a2 b1 c0 11  3  45
   a2 b1 c1 12  3  60;;
  ods rtf file='example.rtf';
  run;
  proc anova;
  class rep A B C;
  model yield=rep A B C A*B A*C B*C A*B*C;
```
means A B C/lsd;
means A*B A*C B*C;
title 'RCBD with a 3x2x2 Factorial Arrangement';
run;
ods rtf close;
run;
RCBD with a 3x2x2 Factorial Arrangement

The ANOVA Procedure

Dependent Variable: Yield

<table>
<thead>
<tr>
<th>Class Level Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Rep</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

Number of Observations Read 36
Number of Observations Used 36
### RCBD with a 3x2x2 Factorial Arrangement

#### The ANOVA Procedure

**Dependent Variable: Yield**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>13</td>
<td>3871.416667</td>
<td>297.801282</td>
<td>63.40</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>22</td>
<td>103.333333</td>
<td>4.696970</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>35</td>
<td>3974.750000</td>
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<table>
<thead>
<tr>
<th>R-Square</th>
<th>Coeff Var</th>
<th>Root MSE</th>
<th>Yield Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.974003</td>
<td>6.550880</td>
<td>2.167249</td>
<td>33.08333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Anova SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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</thead>
<tbody>
<tr>
<td>Rep</td>
<td>2</td>
<td>1238.000000</td>
<td>619.000000</td>
<td>131.79</td>
<td>&lt;.0001</td>
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<tr>
<td>A</td>
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<td>1587.500000</td>
<td>793.750000</td>
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<td>&lt;.0001</td>
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<tr>
<td>B</td>
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<td>140.027778</td>
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<tr>
<td>A*C</td>
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<td>47.388889</td>
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<tr>
<td>B*C</td>
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<td>8.027778</td>
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<td>A<em>B</em>C</td>
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<td>8.388889</td>
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</table>
RCBD with a 3x2x2 Factorial Arrangement

The ANOVA Procedure

Dependent Variable: Yield

Note This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha</td>
<td>0.05</td>
</tr>
<tr>
<td>Error Degrees of Freedom</td>
<td>22</td>
</tr>
<tr>
<td>Error Mean Square</td>
<td>4.69697</td>
</tr>
<tr>
<td>Critical Value of t</td>
<td>2.07387</td>
</tr>
<tr>
<td>Least Significant Difference</td>
<td>1.8349</td>
</tr>
</tbody>
</table>

Means with the same letter are not significantly different.

<table>
<thead>
<tr>
<th>t Grouping</th>
<th>Mean</th>
<th>N</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>41.4167</td>
<td>12</td>
<td>a2</td>
</tr>
<tr>
<td>B</td>
<td>32.6667</td>
<td>12</td>
<td>a1</td>
</tr>
<tr>
<td>C</td>
<td>25.1667</td>
<td>12</td>
<td>a0</td>
</tr>
</tbody>
</table>
**RCBD with a 3x2x2 Factorial Arrangement**

**The ANOVA Procedure**

*t Tests (LSD) for Yield*

*Note* This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

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<tr>
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<td>1.4982</td>
</tr>
</tbody>
</table>

Means with the same letter are not significantly different.

<table>
<thead>
<tr>
<th>t Grouping</th>
<th>Mean</th>
<th>N</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>35.056</td>
<td>18</td>
<td>b1</td>
</tr>
<tr>
<td>B</td>
<td>31.111</td>
<td>18</td>
<td>b0</td>
</tr>
</tbody>
</table>
RCBD with a 3x2x2 Factorial Arrangement

The ANOVA Procedure

Dependent Variable: Yield

Note: This test controls the Type I comparisonwise error rate, not the experimentwise error rate.

<table>
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<tr>
<th>Alpha</th>
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</tr>
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<tbody>
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<td>22</td>
</tr>
<tr>
<td>Error Mean Square</td>
<td>4.69697</td>
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<tr>
<td>Critical Value of t</td>
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</tbody>
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<thead>
<tr>
<th>t Grouping</th>
<th>Mean</th>
<th>N</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>37.8889</td>
<td>18</td>
<td>c1</td>
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<tr>
<td>B</td>
<td>28.2778</td>
<td>18</td>
<td>c0</td>
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## RCBD with a 3x2x2 Factorial Arrangement

### The ANOVA Procedure

#### t Tests (LSD) for Yield

<table>
<thead>
<tr>
<th>Level of A</th>
<th>Level of B</th>
<th>N</th>
<th>Yield Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>a0</td>
<td>b0</td>
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<td>22.5000000</td>
<td>6.6558245</td>
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<tr>
<td>a0</td>
<td>b1</td>
<td>6</td>
<td>27.8333333</td>
<td>8.1342896</td>
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<tr>
<td>a1</td>
<td>b0</td>
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<td>a1</td>
<td>b1</td>
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<td>9.6695398</td>
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<td>b1</td>
<td>6</td>
<td>43.3333333</td>
<td>11.1295403</td>
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<table>
<thead>
<tr>
<th>Level of A</th>
<th>Level of C</th>
<th>N</th>
<th>Yield Mean</th>
<th>Std Dev</th>
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<tbody>
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<table>
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<th>Level of C</th>
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<th>Std Dev</th>
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<td>b0</td>
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