PlSc 724 - FACTORIAL EXPERIMENTS

Factor - refers to a kind of treatment.
Factors will be referred to with capital letters.

Level - refers to several treatments within any factor.
Levels will be referred to with lower case letters.

A combination of lower case letters and subscript numbers will be used to designate individual treatments (a_0, a_1, b_o, b_1, a_0b_0, a_0b_1, etc.)

Experiments and examples discussed so far in this class have been one factor experiments.

For one factor experiments, results obtained are applicable only to the particular level in which the other factor(s) was maintained.

Example: Five seeding rates and one cultivar.

<u>**A factorial is not a design but an arrangement**</u>**.**

A factorial is a study with two or more factors in combination.

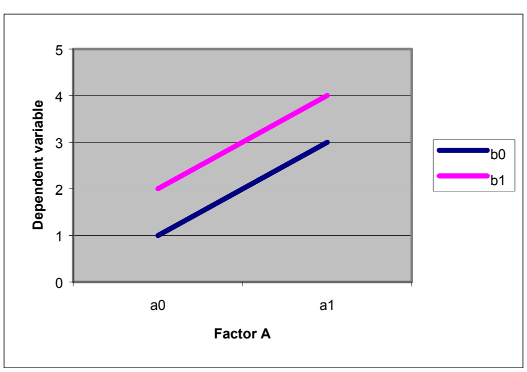
Each level of a factor **must** appear in combination with <u>all</u> levels of the other factors.

Factorial arrangements allow us to study the **interaction** between two or more factors.

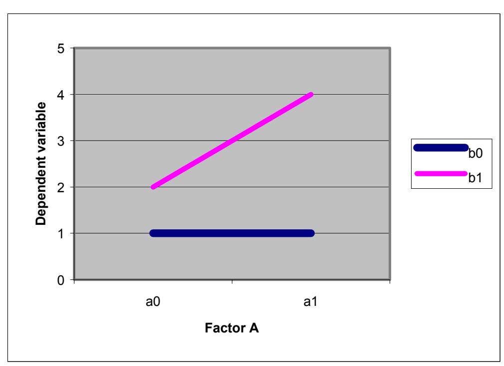
Interaction – 1) the failure for the response of treatments of a factor to be the same for each level of another factor.

2) When the **simple effects** of a factor differ by more than can be attributed to chance, the differential response is called an interaction.

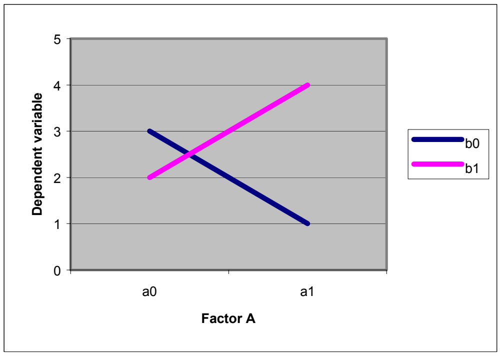
<u>Examples of Interactions</u>

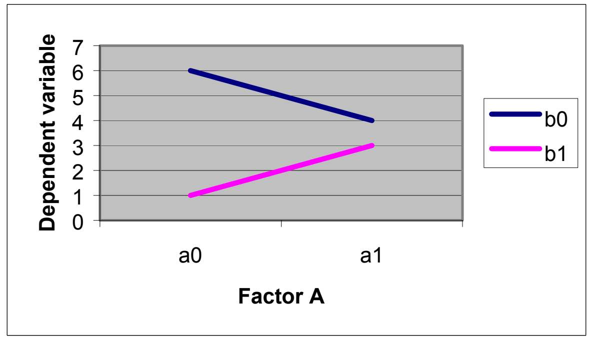


No interaction (similar response)



Interaction (diverging response)





Interaction (crossover response) **Interaction (converging response)**

Simple Effects, Main Effects, and Interactions

Simple effects, main effects, and interactions will be explained using the following data set:

Table 1. Effect of two N rates of fertilizer on grain yield (Mg/ha) of two barley cultivars.

	Fertilizer Rate (B)	
Cultivar (A)	0 kg N/ha (b_0)	60 kg N/ha (b_1)
Larker (a_0)	1.0 (a_0b_0)	3.0 (a_0b_1)
Morex (a_1)	2.0 (a_1b_0)	4.0 (a_1b_1)

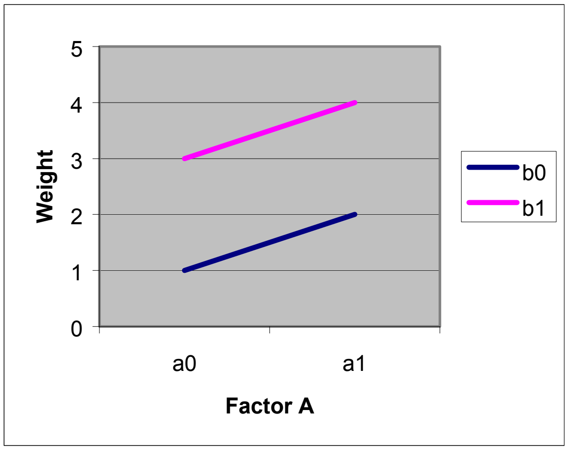
The **simple effect** of a factor is the difference between its two levels at a given level of the other factor.

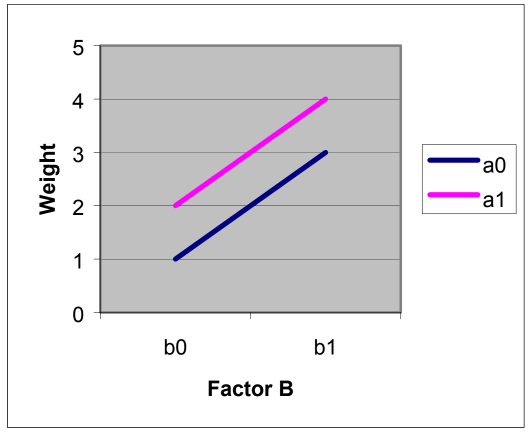
Simple effect of A at $b_0 = a_1b_0 - a_0b_0$
$= 2 - 1$
$=$ **1**

Simple effect of A at $b_1 = a_1b_1 - a_0b_1$
$= 4 - 3$
$=$ **1**

Simple effect of B at $a_0 = a_0b_1 - a_0b_0$
$= 3 - 1$
$=$ **2**

Simple effect of B at $a_1 = a_1b_1 - a_1b_0$
$= 4 - 2$
$=$ **2**





The **main effect** of a factor is the average of the simple effects of that factor over all levels of the other factor.

$$\text{Main effect of A} = \frac{(\text{simple effect of A at } b_0 + \text{simple effect of A at } b_1)}{2}$$
$$= (1 + 1)/2$$
$$= \mathbf{1}$$

$$\text{Main effect of B} = \frac{(\text{simple effect of B at } a_0 + \text{simple effect of B at } a_1)}{2}$$
$$= (2 + 2)/2$$
$$= \mathbf{2}$$

The **interaction** is a function of the difference between the simple effects of A at the two levels of B divided by two, or vice-versa.

(This works only for 2 x 2 factorials)

$$\text{A x B} = 1/2(\text{Simple effect of A at } b_1 - \text{Simple effect of A at } b_0)$$
$$= 1/2(1 - 1)$$
$$= \mathbf{0}$$

or

$$\text{A x B} = 1/2(\text{Simple effect of B at } a_1 - \text{Simple effect of B at } a_0)$$
$$= 1/2(2 - 2)$$
$$= \mathbf{0}$$

Example with an interaction:

Table 2. Effect of two N rates of fertilizer on grain yield (Mg/ha) of two barley cultivars.

	Fertilizer Rate (B)	
Cultivar (A)	0 kg N/ha (b_0)	60 kg N/ha (b_1)
Larker (a_0)	1.0 (a_0b_0)	1.0 (a_0b_1)
Morex (a_1)	2.0 (a_1b_0)	4.0 (a_1b_1)

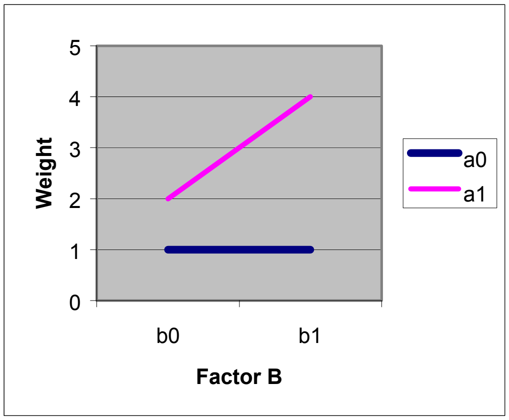
The **simple effect** of a factor is the difference between its two levels at a given level of the other factor.

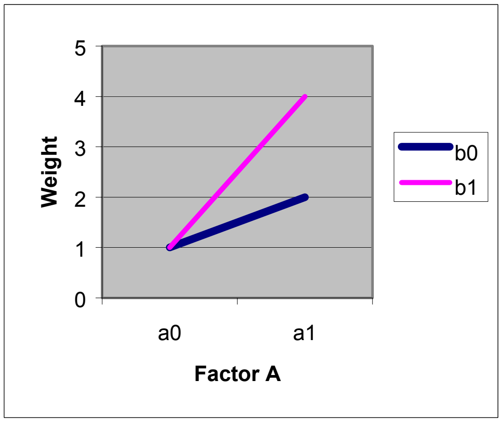
$$\text{Simple effect of A at } b_0 = a_1b_0 - a_0b_0 = 2 - 1 = \mathbf{1}$$

$$\text{Simple effect of A at } b_1 = a_1b_1 - a_0b_1 = 4 - 1 = \mathbf{3}$$

$$\text{Simple effect of B at } a_0 = a_0b_1 - a_0b_0 = 1 - 1 = \mathbf{0}$$

$$\text{Simple effect of B at } a_1 = a_1b_1 - a_1b_0 = 4 - 2 = \mathbf{2}$$





The **main effect** of a factor is the average of the simple effects of that factor over all levels of the other factor.

$$\text{Main effect of A} = \frac{(\text{simple effect of A at } b_0 + \text{simple effect of A at } b_1)}{2}$$
$$= (1 + 3)/2$$
$$= \mathbf{2}$$

$$\text{Main effect of B} = \frac{(\text{simple effect of B at } a_0 + \text{simple effect of B at } a_1)}{2}$$
$$= (0 + 2)/2$$
$$= \mathbf{1}$$

The **interaction** is a function of the difference between the simple effects of A at the two levels of B divided by two, or vice-versa.

(This works only for 2 x 2 factorials)

A x B = 1/2(Simple effect of A at b_1 - Simple effect of A at b_0)
= 1/2(3 - 1)
= **1**

or

A x B = 1/2(Simple effect of B at a_1 - Simple effect of B at a_0)
= 1/2(2 - 0)
= **1**

Facts to Remember about Interactions

1. An interaction between two factors can be measured only if the two factors are tested together in the same experiment.

2. When an interaction is absent, the simple effect of a factor is the same for all levels of the other factors and equals the main effect.

3. When interactions are present, the simple effect of a factor changes as the level of the other factor changes. Therefore, the main effect is different from the simple effects.

Example of ANOVA for a 2x2 Factorial

Table 1. Data for the RCBD analysis of a 2 x 2 factorial arrangement.

	Treatments				
Replicate	a_0b_0	a_0b_1	a_1b_0	a_1b_1	$Y_{.j}$
1	12	19	29	32	92
2	15	22	27	35	99
3	14	23	33	38	108
4	13	21	30	37	101
$Y_{i.}$	54	85	119	142	400=Y..

Step 1. Calculate Correction Factor

$$CF = \frac{Y_{..}^2}{rab}$$

$$= \frac{400^2}{4*2*2}$$

$$= 10,000$$

Step 2. Calculate Total SS

$$\text{Total SS} = \sum Y_{ij}^2 - CF$$

$$= (12^2 + 15^2 + 14^2 + ... + 37^2) - CF$$
$$= 1,170.0$$

Step 3. Calculate Replicate SS

$$\text{Rep}SS = \frac{\sum Y_{.j}^2}{ab} - CF$$

$$= \frac{(92^2 + 99^2 + 108^2 + 101^2)}{2*2} - CF$$

$$= 32.5$$

Step 4. Partition Treatment SS

Step 4.1. Make Table of Treatment Totals

Table . Table of treatment totals.

	a_0	a_1	$\sum B$
b_0	54	119	173
b_1	85	142	227
$\sum A$	139	261	400

Step 4.2. Calculate A SS

$$\text{A SS} = \frac{\sum \text{A}^2}{rb} - CF$$

$$= \frac{(139^2 + 261^2)}{4*2} - CF$$

$$= 930.25$$

Step 4.3. Calculate B SS

$$\text{B SS} = \frac{\sum \text{B}^2}{ra} - CF$$

$$= \frac{(173^2 + 227^2)}{4*2} - CF$$

$$= 182.25$$

Step 4.4. Calculate A x B SS

$$\text{AxB SS} = \frac{\sum \text{ab}^2}{r} - \text{CF} - \text{A SS} - \text{B SS}$$

$$= \frac{(54^2 + 85^2 + 119^2 + 142^2)}{4} - \text{CF} - \text{A SS} - \text{B SS}$$

$$= 4.0$$

NOTE: A SS + B SS + A x B SS = Treatment SS

Step 5. Calculate Error SS

Error SS = Total SS - Rep SS - A SS - B SS - A x B SS

= 21.0

Step 6. Do the ANOVA

SOV	df	SS	MS	F (assuming A and B fixed)
Rep	r - 1 = 3	32.5	10.833	Rep MS/Error MS = 4.64*
A	a - 1 = 1	930.25	930.25	A MS/Error MS = 398.679**
B	b - 1 = 1	182.25	182.25	B MS/Error MS = 78.107**
A x B	(a - 1)(b - 1) = 1	4.00	4.00	AxB MS/Error MS = 1.714
Error	(r - 1)(ab - 1) = 9	21.00	2.333	
Total	rab - 1 =15	1170.00		

Step 7. Calculate LDS's (0.05)

Step 7.1 Calculate LSD_A

$$\text{LSD A} = t_{.05/2;9df}\sqrt{\frac{2\text{Error MS}}{rb}}$$

$$= 2.262\sqrt{\frac{2(2.333)}{4*2}}$$

$$= 1.7$$

Mean of treatment A averaged across all levels of B.

Treatment	Mean
a_0	17.4 a
a_1	32.6 b

Step 7.2 Calculate LSD_B

$$\text{LSD B} = t_{.05/2;9df}\sqrt{\frac{2\text{Error MS}}{ra}}$$

$$= 2.262\sqrt{\frac{2(2.333)}{4*2}}$$

$$= 1.7$$

Mean of treatment B averaged across all levels of A.

Treatment	Mean
b_0	21.6 a
b_1	28.4 b

Step 7.3 Calculate $LSD_{A \times B}$

$$LSD\ AxB = t_{.05/2;9df}\sqrt{\frac{2\text{Error MS}}{r}}$$

$$= 2.262\sqrt{\frac{2(2.333)}{4}}$$

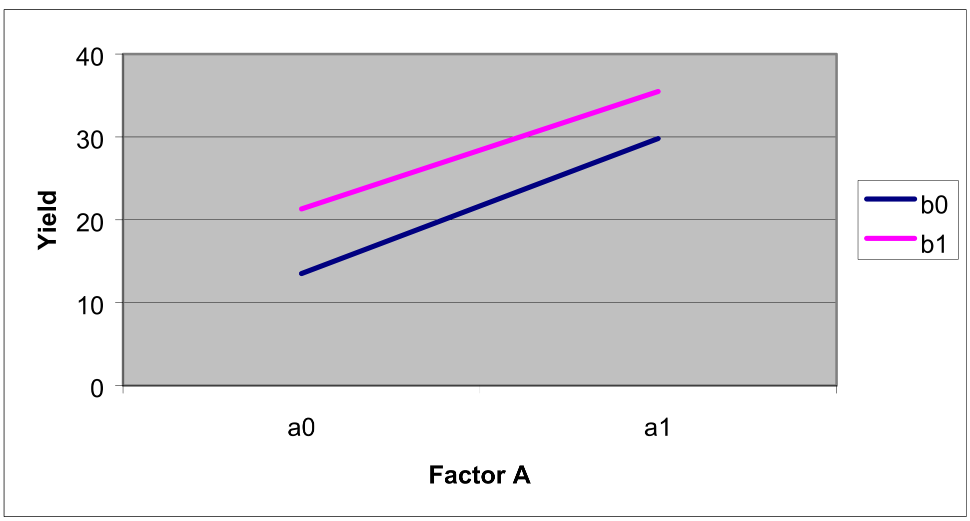
$$= 2.4$$

Mean of the interaction of A and B.

Treatment	Mean
a_0b_0	13.5 a
a_0b_1	21.3 a
a_1b_0	29.8 a
a_1b_1	35.5 a

Mean of the interaction of A and B.

	Factor B	
Factor A	b_0	b_1
a_0	13.5 a	21.3 a
a_1	29.8 a	35.5 a



You can see from the figure above that the two lines are nearly parallel. This indicates that B is responding similarly at all levels of A; thus, there is no interaction.

Example of a CRD with a 4x3 Factorial Arrangement

Given there are 3 replicates, the SOV and df would be as follows:

SOV	Df
A	a-1 = 3
B	b–1 = 2
AxB	(a-1)(b-1) = 6
Error	By subtraction = 24
Total	rab-1 = 35

Example of a Latin Square with a 3x2 Factorial Arrangement

What would be the size of the Latin Square?

Answer: 6

The six treatments would be all combinations of A and B (a_0b_0, a_1b_0, a_2b_0, a_0b_1, a_1b_1, a_2b_1).

The ANOVA table would be as follows:

SOV	Df
Row	ab-1 = 5
Column	ab-1 = 5
A	a-1 = 2
B	b-1 =1
AxB	(a-1)(b-1) = 2
Error	(ab-1)(ab-2) = 20
Total	$(ab)^2$ –1=35

Note that r=ab

Example of a RCBD with a 4x3x2 Arrangement

Given there are 5 replicates, the ANOVA would look as follows:

SOV	Df
Rep	r-1 = 4
A	a-1 = 3
B	b-1 = 2
C	c-1 = 1
AxB	(a-1)(b-1) = 6
AxC	(a-1)(c-1) = 3
BxC	(b-1)(c-1) = 2
AxBxC	(a-1)(b-1)(c-1) = 6
Error	(r-1)(abc-1) = 92
Total	rabc-1 = 119

In order to calculate the Sums of Squares for A, B, C, AxB, Ax C, BxC, and AxBxC, you will need to make several tables of treatment totals.

The general outline of these tables is as follows:

Table 1. Totals used to calculate A SS, B SS, and AxB SS.

	a_0	a_1	a_2	a_3	$\sum B$
b_0	a_0b_0	a_1b_0			
b_1					
b_2					
$\sum A$					

Remember AxB SS = $\dfrac{\sum(ab)^2}{rc}$ **- CF – A SS – B SS**

Table 2. Totals used to calculate A SS, C SS, and AxC SS.

	a_0	a_1	a_2	a_3	$\sum C$
c_0	a_0c_0	a_1c_0			
c_1					
$\sum A$					

Remember AxC SS = $\dfrac{\sum(ac)^2}{rb}$ **- CF – A SS – C SS**

Table 3. Totals used to calculate B SS, C SS, and BxC SS.

	b_0	b_1	b_2	$\sum C$
c_0	b_0c_0	b_1c_0		
c_1				
$\sum B$				

Remember BxC SS = $\dfrac{\sum(bc)^2}{ra}$ **- CF – B SS – C SS**

Table 4. Values used to calculate Total SS, Rep SS, and AxBxC SS.

	Rep 1	Rep 2	Rep 3	$\sum ABC$
$a_0b_0c_0$				
$a_0b_0c_1$				
$a_0b_1c_0$				
…				
$a_3b_1c_1$				
$\sum$Rep				

Remember AxBxC SS = $\dfrac{\sum(abc)^2}{r}$ **- CF – A SS - B SS – C SS – AxB SS – AxC SS – BxC SS**

Linear Model

$Y_{ijk} = \mu + \nu_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$

Where:
μ = Experiment mean
ν_i = Rep effect if the i^{th} replicate
α_j = Effect of the j^{th} level of factor A
β_k = Effect of the k^{th} level of factor B
$(\alpha\beta)_{jk}$ = A x B interaction effect
ϵ_{ijk} = Random error

Advantages of Factorial Arrangements

1. Provides estimates of interactions.
2. Possible increase in precision due to so-called "hidden replication."
3. Experimental rates can be applied over a wider range of conditions.

Disadvantages of Factorial Arrangements

1. Some treatment combinations may be of little interest.
2. Experimental error may become large with a large number of treatments.
3. interpretation may be difficult (especially for 3-way or more interactions).

Randomizing Factorial Arrangements

1. Assign numbers to treatment combinations.
2. Randomize treatments according to design.

<u>Example - RCBD with a 2x4 Factorial Arrangement</u>

Treatment	Treatment number	Treatment	Treatment number
a_0b_0	1	a_1b_0	5
a_0b_1	2	a_1b_1	6
a_0b_2	3	a_1b_2	7
a_0b_3	4	a_1b_3	8

Rep 1	3 a_0b_2	7 a_1b_2	2 a_0b_1	6 a_1b_1	4 a_0b_3	5 a_1b_0	1 a_0b_0	8 a_1b_3

Interpreting Results of ANOVA Involving Interaction Terms

Interpretation should always begin with the higher level interaction terms (e.g. three-way interactions before two-interactions, etc.).

Interpretation of the main effects should **<u>never</u>** be done before interpreting the interaction terms.

The F-test for interaction terms can be significant because of two reasons.

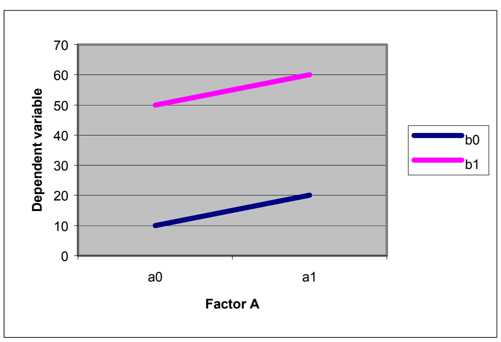
1. True interaction.

2. Differences in magnitude between treatment means.

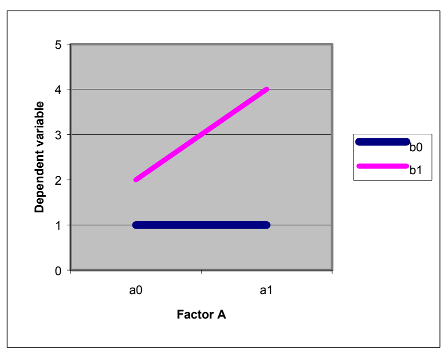
Differences in magnitude when responses are similar, but the differences between means are great.

An example of this would be yield of a series of varieties having the same relative rank at two locations that differ greatly in yield due to differences in growing conditions (e.g. favorable growing conditions vs. drought conditions).

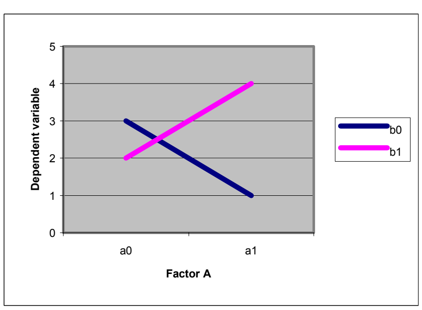
<u>Examples of Interactions and Differences in Magnitude</u>



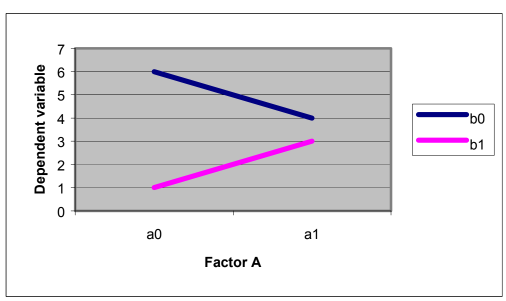
Differences in magnitude



Interaction



Interaction



Interaction

Flow Chart for Interpreting ANOVA's with Interaction Terms