Split Block Arrangement

The split block arrangement is especially suited for two factor experiments in which the desired precision for measuring the interaction effect between two factors is greater than either of the two factors.

This is accomplished with the use of three plot sizes:

1. Horizontal strip for the first factor
2. Vertical strip for the second factor
3. Intersection plot for the interaction

<u>Example of Split Block Arrangement Layout</u>

Factor A = vertical factor, 4 levels
Factor B = horizontal factor, 2 levels
Replicates = 3

Replicate 1

a_0	a_1	a_3	a_2
b_1	b_1	b_1	b_1
b_0	b_0	b_0	b_0

Replicate 2

a_3	a_1	a_2	a_0
b_1	b_1	b_1	b_1
b_0	b_0	b_0	b_0

Replicate 3

a_3	a_0	a_2	a_1
b_0	b_0	b_0	b_0
b_1	b_1	b_1	b_1

<u>Determining Which Factor to Use as the Horizontal and/or Vertical Factor</u>

In general, it does not matter which factor you choose to use as the horizontal or vertical factor.

Randomization Procedure

1. Randomly assign treatments to horizontal strips.
2. Randomly assign treatments to vertical strips.

Expected Mean Squares for the Split Block Arrangement

The example to be given will be for an RCBD with factor A as the horizontal factor and factor B as the vertical factor. Factor A and B will be considered fixed effects.

Source of variation	df	Expected mean square
Replicate	r-1	$\sigma^2 + a\sigma^2_\theta + b\sigma^2_\gamma + ab\sigma^2_R$
A (horizontal factor)	a-1	$\sigma^2 + b\sigma^2_\gamma + r\sigma^2_{AB} + rb\sigma^2_A$
Error (a) = Rep x A	(r-1)(a-1)	$\sigma^2 + b\sigma^2_\gamma$
B (vertical factor)	b-1	$\sigma^2 + a\sigma^2_\theta + r\sigma^2_{AB} + ra\sigma^2_B$
Error (b) = Rep x B	(r-1)(b-1)	$\sigma^2 + a\sigma^2_\theta$
AxB	(a-1)(b-1)	$\sigma^2 + r\sigma^2_{AB}$
Error (c) = Rep x A X B	(r-1)(a-1)(b-1)	σ^2
Total	rab-1	

ANOVA of a Split Block Arrangement

		Vertical Whole Plots (B_k)				
		b_0	b_1	b_2	b_3	Horizontal Plot totals
Replicate 1	a_0	13.8	15.5	21.0	18.9	$69.2 = Y_{11.}$
	a_1	19.3	22.2	25.3	25.9	$92.7 = Y_{21.}$
Vertical Plot Total $Y_{.1k}$		33.1	37.7	46.3	44.8	$161.9 = Y_{.1.}$
Replicate 2	a_0	13.5	15.0	22.7	18.3	$69.5 = Y_{12.}$
	a_1	18.0	24.2	24.8	26.7	$93.7 = Y_{22.}$
Vertical Plot Total $Y_{.2k}$		31.5	39.2	47.5	45.0	$163.2 = Y_{.2.}$
Replicate 3	a_0	13.2	15.2	22.3	19.6	$70.3 = Y_{13.}$
	a_1	20.5	25.4	28.4	27.6	$101.9 = Y_{23.}$
Vertical Plot Total $Y_{.3k}$		33.7	40.6	50.7	47.2	$172.2 = Y_{.3.}$

Treatment Totals Table

	b_0	b_1	b_2	b_3	$\Sigma A_{i..}$
a_0	40.5	45.7	66.0	56.8	209.0
a_1	57.8	71.8	78.5	80.2	288.3
$\Sigma B_{..k}$	98.3	117.5	144.5	137.0	497.3

Step 1. Calculate Correction Factor:

$$\mathrm{CF} = \frac{\mathrm{Y}_{...}^2}{\mathrm{rxaxb}} = \frac{(497.3)^2}{3x2x4} = 10,304.47$$

Step 2. Calculate Total SS

$$\mathrm{Total\,SS} = \sum \mathrm{Y}_{\mathrm{ijk}}^2 - CF$$

$$= (13.8^2 + 19.3^2 + 13.5^2 + ... + 27.6^2) - CF$$

$$= 516.2$$

Step 3. Calculate Replicate SS

$$\mathrm{Rep\ SS} = \frac{\sum \mathrm{Y}_{.\mathrm{j}.}^2}{axb} - CF$$

$$= \frac{(161.9^2 + 163.2^2 + 172.2^2)}{(2x4)} - CF$$

$$= 7.87$$

Step 4. Calculate A SS

$$\mathrm{A\ SS} = \frac{\sum \mathrm{Y}_{\mathrm{i}..}^2}{\mathrm{rxb}} - CF$$

$$= \frac{(209.0^2 + 288.3^2)}{(3x4)} - CF$$

$$= 262.02$$

Step 5. Calculate Horizontal Whole Plot SS

$$\text{Horizontal Whole Plot SS} = \frac{\sum Y_{ij.}^2}{b} - CF$$

$$= \frac{(69.2^2 + 92.7^2 + ... + 101.9^2)}{4} - CF$$

$$= 274.92$$

Step 6. Calculate Whole Plot Error SS = Error(a) SS

Horizontal Whole Plot SS - A SS - Rep SS = **5.03**

Step 7. Calculate B SS

$$\text{B SS} = \frac{\sum Y_{..k}^2}{ra} - CF$$

$$= \frac{(98.3^2 + 117.5^2 + 144.5^2 + 137.0^2)}{(3x2)} - CF$$

$$= 215.26$$

Step 8. Calculate Vertical Whole Plot SS

$$\text{Vertical Whole Plot SS} = \frac{\sum Y_{.jk}^2}{a} - CF$$

$$= \frac{(33.1^2 + 37.7^2 + ... + 47.2^2)}{2} - CF$$

$$= 225.605$$

Step 9. Calculate Vertical Whole Plot Error SS = Error(b) SS

Vertical Whole Plot SS - B SS - Rep SS = **2.475**

Step 10. Calculate AxB SS

$$\text{AxB SS} = \frac{\sum Y_{i.k}^2}{r} - \text{CF - A SS - B SS}$$

$$= \frac{(40.5^2 + 45.7^2 + ... + 80.2^2)}{3} \text{ - CF - A SS - B SS}$$

$$= 18.7$$

Step 11. Calculate Error(c) SS = Total SS - Rep SS - A SS - Error(a) SS - B SS - Error(b) SS - AxB SS = =**4.765**

Step 12. Make ANOVA Table (Assuming A and B are fixed effects)

SOV	df	SS	MS	F (A and B fixed)
Replicate	2	7.87	3.935	
A	1	262.02	262.02	A MS/Error(a) MS = 104.183**
Error(a)	2	5.03	2.515	
B	3	215.26	71.753	B MS/Error(b) MS = 174.158**
Error(b)	6	2.475	0.412	
AxB	3	18.70	6.233	AxB MS/Error(c) MS = 7.850*
Error(c)	6	4.765	0.794	
Total	23	516.12		

LSD's for Split Block Arrangement

1. To compare two horizontal means averaged over all vertical treatments (.e.g. a_0 vs a_1)

$$t_{\alpha/2,\text{Error(a) df}}\sqrt{\frac{2Error(a)MS}{rb}}$$

$$= 4.303\sqrt{\frac{2(2.5150)}{3X4}}$$

$$= 2.79$$

2. To compare two vertical means averaged over all horizontal treatments (.e.g. b_0 vs b_1)

$$t_{\alpha/2,\text{Error(b) df}}\sqrt{\frac{2Error(b)MS}{ra}}$$

$$= 4.303\sqrt{\frac{2(0.412)}{3X2}}$$

$$= 1.59$$

3. To compare two horizontal means at the same level of the vertical factor (e.g. a_0b_0 vs a_1b_0)

$$t_{ac'}\sqrt{\frac{2[(b-1)Error(c)MS + Error(a)MS]}{rb}}$$

and

$$t_{ac'} = \frac{(b-1)Error(c)MS(t_{\alpha/2,Error(c)df}) + Error(a)MS(t_{\alpha/2,Error(a)df})}{(b-1)Error(c)MS + Error(a)MS}$$

$\therefore$

$$t_{ac'} = \frac{(4-1)(0.794)(2.447) + 2.515(4.303)}{(4-1)0.794 + 2.515} = 3.4$$

and LSD $= 3.4\sqrt{\frac{2[(4-1)0.794 + 2.515]}{3x4}}$

$= 3.072$

4. To compare two vertical means at the same level of the horizontal factor (e.g. a_0b_0 vs a_0b_1)

$$t_{bc'}\sqrt{\frac{2[(a-1)Error(c)MS + Error(b)MS]}{ra}}$$

and

$$t_{bc'} = \frac{(a-1)Error(c)MS(t_{\alpha/2,Error(c)df}) + Error(b)MS(t_{\alpha/2,Error(b)df})}{(a-1)Error(c)MS + Error(b)MS}$$

$\therefore$

$$t_{bc'} = \frac{(2-1)(0.794)(2.447) + 0.412(2.447)}{(2-1)0.794 + 0.412} = 2.449$$

$$\text{and LSD} = 2.447\sqrt{\frac{2[(2-1)0.794 + 0.412]}{3x2}}$$

$$= 1.553$$

5. To compare two vertical means at different levels of the horizontal factor (e.g. a_0b_0 vs a_1b_3)

$$t_{abc'}\sqrt{\frac{2[(a-1)(b-1)Error(c)MS + (b-1)Error(b)MS + (a-1)Error(a)MS]}{rab}}$$

and

$$t_{abc'} = \frac{(a-1)(b-1)Error(c)MS(t_{\alpha/2,Error(c)df}) + (b-1)Error(b)MS(t_{\alpha/2,Error(b)df}) + (a-1)Error(a)MS(t_{\alpha/2,Error(a)df})}{(a-1)(b-1)Error(c)MS + (b-1)Error(b)MS + (a-1)Error(a)MS}$$

$\therefore$

$$t)_{abc'} = \frac{(2-1)(4-1)(0.794)(2.447) + (4-1)0.412(2.447) + (2-1)2.515(4.303)}{(2-1)(4-1)0.794 + (4-1)0.412 + (2-1)2.515} = 3.208$$

$$\text{and LSD} = 3.208\sqrt{\frac{2[(2-1)(4-1)0.794 + (4-1)0.412 + (2-1)2.515]}{3x2x4}}$$

$$= 2.293$$