SPLIT PLOT IN TIME AND SPACE AND COMBINED ANALYSES

Split Plot in Time

This arrangement occurs when you have an experiment where you collect data from the same experimental unit over a series of dates.

An example of this would be an experiment that includes a perennial species (e.g. alfalfa) and different harvest dates or an experiment where you measure water use each week in a field experiment.

The ANOVA for this situation is similar to a split block.

Example

An experiment comparing the yield of 10 alfalfa cultivars cut at three different times during the growing season. Cultivars and cutting times are fixed effects.

SOV	Df	Expected Mean Squares	F-test
Rep	r-1	$\sigma^2 + c\sigma^2_\Theta + t\sigma^2_\delta + ct\sigma^2_R$	
Cultivar	c-1	$\sigma^2 + t\sigma^2_\delta + rt\frac{\sum \gamma^2}{c-1}$	C MS/Error(a) MS
Error(a)	(r-1)(c-1)	$\sigma^2 + t\sigma^2_\delta$	
Time	t-1	$\sigma^2 + c\sigma^2_\Theta + rc\frac{\sum \tau^2}{t-1}$	T MS/Error(b) MS
Error(b)	(r-1)(t-1)	$\sigma^2 + c\sigma^2_\Theta$	
Cultivar*Time	(c-1)(t-1)	$\sigma^2 + r\frac{\sum (\gamma\tau)^2}{(c-1)(t-1)}$	C*T MS/Error(c) MS
Error(c)	(r-1)(c-1)(t-1)	σ^2	
Total	rct-1		

Split Plot in Time and Space

At times you may have an experiment where you are collecting data from experimental units at different times and from different depths.

An example of this situation would be collecting water use data over time from different depths.

Example

You are to measure water use by different sunflower hybrids at two week intervals. At each measurement time, you will collect water measures at one foot intervals from 1 to 4 feet.

SOV	Df	F-test
Replicate	r-1	
Hybrid	h-1	H MS/Error(a) MS
Error(a)	(r-1)(h-1)	
Time	(t-1)	T MS/Error(b) MS
Error(b)	(r-1)(t-1)	
Hybrid*Time	(h-1)(t-1)	H*T MS/Error(c) MS
Error(c)	(r-1)(h-1)(t-1)	
Depth	(d-1)	D MS/Error(d) MS
Error(d)	(r-1)(d-1)	
Hybrid*Depth	(h-1)(d-1)	H*D MS/Error(e) MS
Error(e)	(r-1)(h-1)(d-1)	
Time *Depth	(t-1)(d-1)	T*D MS/Error(f) MS
Hybrid*Time*Depth	(h-1)(t-1)(d-1)	H*T*D MS/Error(f) MS
Error(f)	By subtraction	
Total	rhtd-1	

Combining Similar Experiments

Before performing combined analysis across locations, runs, years, etc., you need to make sure the error mean squares are homogeneous using Bartlett's Chi-square test if you have more than two mean squares.

If you have two mean squares to test, you can use an F-test (i.e. Larger MS/Smaller MS).

F-tests on location, year, and run are generally not valid since these sources of variation are not replicated.

1. Combined CRD (treatments fixed, locations random).

SOV	Df	Expected Mean Square	F-test
Location	l-1	--	Non-valid
Treatment	t-1	$\sigma^2 + r\sigma^2_{LT} + rl\frac{\sum \tau^2}{t-1}$	T MS/Loc*Trt MS
Location*Treatment	(l-1)(t-1)	$\sigma^2 + r\sigma^2_{LT}$	Loc*Trt MS/Error MS
Error	lt(r-1)	σ^2	
Total	lrt-1		

2. Combined RCBD (treatments fixed, locations random).

SOV	Df	Expected Mean Square	F-test
Location	l-1	--	Non-valid
Rep(Loc)	l(r-1)	--	Non-valid
Treatment	t-1	$\sigma^2 + r\sigma^2_{LT} + rl\frac{\sum \tau^2}{t-1}$	T MS/Loc*Trt MS
Location*Treatment	(l-1)(t-1)	$\sigma^2 + r\sigma^2_{LT}$	Loc*Trt MS/Error MS
Error	l(r-1)(t-1)	σ^2	
Total	lrt-1		

3. Combined RCBD (treatments fixed, locations and year random).

SOV	Df	Expected Mean Square	F-test
Year	y-1		Non-valid
Location	l-1	$\sigma^2 + rt\sigma^2_{YL} + yrt\sigma^2_{L}$	L MS/Y*L MS
Year*Location	(y-1)(l-1)	$\sigma^2 + rt\sigma^2_{YL}$	Non-valid
Rep(Year*Loc)	yl(r-1)	--	Non-valid
Treatment	t-1	$\sigma^2 + r\sigma^2_{YLT} + rl\sigma^2_{YT} + ry\sigma^2_{LT} + rl\frac{\sum \tau^2}{t-1}$	$\frac{\text{T MS + YLT MS}}{\text{YT MS + LT MS}}$
Location*Treatment	(l-1)(t-1)	$\sigma^2 + r\sigma^2_{YLT} + ry\sigma^2_{LT}$	LT MS/YLT MS
Year*Treatment	(y-1)(t-1)	$\sigma^2 + r\sigma^2_{YLT} + rl\sigma^2_{YT}$	YT MS/YLT MS
Year*Loc*Trt	(y-1)(l-1)(t-1)	$\sigma^2 + r\sigma^2_{YLT}$	YLT MS/Error MS
Error	yl(r-1)(t-1)	σ^2	
Total	lrt-1		

To simplify the previous analysis, many people combine or confound locations and years into a source of variation called environment.

The consequences of this decision are that you can no longer test locations using a valid F-test.

The ANOVA would look like:

SOV	Df	Expected Mean Square	F-test
Environment	e-1	--	Non-valid
Rep(Env)	e(r-1)	--	Non-valid
Treatment	t-1	$\sigma^2 + r\sigma^2_{ET} + re\frac{\sum \tau^2}{t-1}$	T MS/Env*Trt MS
Environment*Treatment	(e-1)(t-1)	$\sigma^2 + r\sigma^2_{ET}$	Env*Trt MS/Error MS
Error	e(r-1)(t-1)	σ^2	
Total	ert-1		

Analysis of variance of RCBD with split plot, split-split plot, and split block arrangements, and calculation of LSD values is more complicated than the situations discussed above.

The following reference is an excellent source of information for these situations:

Carmer, S.G., W.E. Nyquist, and W.M. Walker. 1989. Least significant differences for combined analyses of experiments with two-or three-factor treatment designs. Agron. J. 81:665-672.