

Split-split Plot Arrangement

The split-split plot arrangement is especially suited for three or more factor experiments where different levels of precision are required for the factors evaluated.

This arrangement is characterized by:

1. Three plot sizes corresponding to the three factors; namely, the largest plot for the main factor, the intermediate size plot for the subplot factor, and the smallest plot for the sub-subplot factor.
2. There are three levels of precision with the main plot factor receiving the lowest precision, and the sub-subplot factor receiving the highest precision.

Example

Split-split plot arrangement randomized as an RCBD. Three levels of the whole plot factor, A, two levels of the subplot factor, B, and three levels of the sub-subplot factor, C. Diagram shows the first replicate.

a_0 Whole plot	a_1b_0 subplots	$a_2b_1c_0$	$a_2b_1c_2$	$a_2b_1c_1$
	a_1b_1 subplots	$a_2b_0c_1$	$a_2b_0c_0$	$a_2b_0c_2$

Randomization Procedure

The randomization procedure for the split-split plot arrangement consists of three parts:

1. Randomly assign whole plot treatments to whole plots based on the experimental design used.
2. Randomly assign subplot treatments to the subplots.
3. Randomly assign sub-subplot treatments to the sub-subplots.

The experimental design used to randomize the whole plots will not affect randomization of the sub and sub-subplots.

Expected Mean Squares for the Split-split Plot Arrangement

The example to be given will be for an RCBD with factor A as the whole plot factor, factor B as the subplot factor, and factor C as the sup-subplot factor. Factors A, B, and C will be considered random effects.

Source of variation	df	Expected mean square
Replicate	$r-1$	$\sigma^2 + c\sigma_\theta^2 + bc\sigma_\gamma^2 + abc\sigma_R^2$
A	$a-1$	$\sigma^2 + c\sigma_\theta^2 + bc\sigma_\gamma^2 + r\sigma_{ABC}^2 + rb\sigma_{AC}^2 + rc\sigma_{AB}^2 + rbc\sigma_A^2$
Error (a) = RepxA	$(r-1)(a-1)$	$\sigma^2 + c\sigma_\theta^2 + bc\sigma_\gamma^2$
B	$b-1$	$\sigma^2 + c\sigma_\theta^2 + r\sigma_{ABC}^2 + ra\sigma_{BC}^2 + rc\sigma_{AB}^2 + rac\sigma_B^2$
AxB	$(a-1)(b-1)$	$\sigma^2 + c\sigma_\theta^2 + r\sigma_{ABC}^2 + rc\sigma_{AB}^2$
Error (b) = RepxB(A)	$a(r-1)(b-1)$	$\sigma^2 + c\sigma_\theta^2$
C	$c-1$	$\sigma^2 + r\sigma_{ABC}^2 + ra\sigma_{BC}^2 + rb\sigma_{AC}^2 + rab\sigma_C^2$
AxC	$(a-1)(c-1)$	$\sigma^2 + r\sigma_{ABC}^2 + rb\sigma_{AC}^2$
BxC	$(b-1)(c-1)$	$\sigma^2 + r\sigma_{ABC}^2 + ra\sigma_{BC}^2$
AxBxC	$(a-1)(b-1)(c-1)$	$\sigma^2 + r\sigma_{ABC}^2$
Error (c) = RepxC(AxB)	$ab(r-1)(c-1)$	σ^2
Total	$rabc-1$	

ANOVA of a Split-split Plot Arrangement - Table 1. Data for split-split plot example

Treatments			Replicates				Treatment
A _j	B _k	C _l	1	2	3	4	totals
0	0	0	25.7	25.4	23.8	22.0	96.9
0	0	1	31.8	29.5	28.7	26.4	116.4
0	0	2	34.6	37.2	29.1	23.7	124.6
Subplot tot. Y _{i00.}			92.1	92.1	81.6	72.1	337.9= Y _{.00.}
0	1	0	27.7	30.3	30.2	33.2	121.4
0	1	1	38.0	40.6	34.6	31.0	144.2
0	1	2	42.1	43.6	44.6	42.7	173.0
Subplot tot. Y _{i01.}			107.8	114.5	109.4	106.9	438.6= Y _{.01.}
Whole plot tot. Y _{i0.}			199.9	206.6	191.0	179.0	776.5= Y _{.0.}
1	0	0	28.9	24.7	27.8	23.4	104.8
1	0	1	37.5	31.5	31.0	27.8	127.8
1	0	2	38.4	32.5	31.2	29.8	131.9
Subplot tot. Y _{i10.}			104.8	88.7	90.0	81.0	364.5= Y _{.10.}
1	1	0	38.0	31.0	29.5	30.7	129.2
1	1	1	36.9	31.9	31.5	35.9	136.2
1	1	2	44.2	41.6	38.9	37.6	162.3
Subplot tot. Y _{i11.}			119.1	104.5	99.9	104.2	427.7= Y _{.11.}
Whole plot tot. Y _{i1.}			223.9	193.2	189.9	185.2	792.2= Y _{.1.}
2	0	0	23.4	24.2	21.2	20.9	89.7
2	0	1	25.3	27.7	23.7	24.3	101.0
2	0	2	29.8	29.9	24.3	23.8	107.8
Subplot tot. Y _{i20.}			78.5	81.8	69.2	69.0	298.5= Y _{.20.}
2	1	0	20.8	23.0	25.2	23.1	92.1
2	1	1	29.0	32.0	26.5	31.2	118.7
2	1	2	36.6	37.8	34.8	40.2	149.4
Subplot tot. Y _{i21.}			86.4	92.8	86.5	94.5	360.2= Y _{.21.}
Whole plot tot. Y _{i2.}			164.9	174.6	155.7	163.5	658.7= Y _{.2.}
Rep total Y _{i...}			588.7	574.4	536.6	527.7	2227.4= Y _{...}

Table 2. Totals for two-way interactions.

A X B (Y _{.jk})			A X C (Y _{.jl})			B X C (Y _{.kl})		
	b ₀	b ₁	c ₀	c ₁	c ₂		b ₀	b ₁
a ₀	337.9	438.6	218.3	260.6	297.6	c ₀	291.4	342.7
a ₁	364.5	427.7	234.0	264.0	294.2	c ₁	345.2	399.1
a ₂	298.5	360.2	181.8	219.7	257.2	c ₂	364.3	484.7

Table 3. Totals for main effects.

A (Y _{.j.})			B (Y _{..k})		C (Y _{..l})		
a ₀	a ₁	a ₂	b ₀	b ₁	c ₀	c ₁	c ₂
776.5	792.2	658.7	1000.9	1226.5	634.1	744.3	849.0

Step 1. Calculate correction factor

$$\begin{aligned}
 CF &= \frac{Y_{...}^2}{rabc} \\
 &= \frac{(2227.4)^2}{(4 \times 3 \times 2 \times 3)} \\
 &= 68,907.094
 \end{aligned}$$

Step 2. Calculate total sum of squares

$$\text{Total SS} = \sum Y_{ijkl}^2 - CF$$

$$\text{Total SS} = (25.7^2 + 25.4^2 + 23.8^2 + \dots + 40.2^2) - CF = 2840.606$$

Step 3. Calculate replicate sum of squares

$$\begin{aligned}\text{Rep SS} &= \frac{\sum Y_{i..}^2}{abc} - CF \\ &= \frac{(588.7^2 + 574.4^2 + 536.6^2 + 527.7^2)}{3 \times 2 \times 3} - CF \\ &= 143.456\end{aligned}$$

Step 4. Calculate A sum of squares.

$$\begin{aligned}\text{A SS} &= \frac{\sum Y_{.j.}^2}{rbc} - CF \\ &= \\ &= \frac{(776.5^2 + 792.2^2 + 658.7^2)}{4 \times 2 \times 3} - CF \\ &= 443.689\end{aligned}$$

Step 5. Calculate Whole plot sum of squares.

$$\begin{aligned}\text{Whole Plot SS} &= \frac{\sum Y_{ij}^2}{bc} - CF \\ &= \frac{(199.9^2 + 206.6^2 + 191.0^2 + \dots + 163.5^2)}{bc} - CF \\ &= 698.903\end{aligned}$$

Step 6. Calculate Error(a) sum of squares.

$$\begin{aligned}\text{Error (a) SS} &= \text{Whole plot SS} - \text{A SS} - \text{Rep SS} \\ &= 698.903 - 443.689 - 143.456 = 111.749\end{aligned}$$

Step 7. Calculate B sum of squares.

$$\begin{aligned} \text{B SS} &= \frac{\sum Y_{.k}^2}{r \cdot c} - \text{CF} \\ &= \frac{(1000.9^2 + 1226.5^2)}{4 \times 3 \times 3} - \text{CF} \\ &= 706.880 \end{aligned}$$

Step 8. Calculate A X B sum of squares.

$$\begin{aligned} \text{AxB SS} &= \frac{\sum Y_{jk}^2}{r \cdot c} - \text{CF} - \text{A SS} - \text{B SS} \\ &= \frac{(337.9^2 + 364.5^2 + 298.5^2 + \dots + 438.6^2)}{4 \times 3} - \text{CF} - \text{A SS} - \text{B SS} \\ &= 40.687 \end{aligned}$$

Step 9. Calculate subplot sum of squares.

$$\begin{aligned} \text{Subplot SS} &= \frac{\sum Y_{ijk}^2}{c} - \text{CF} \\ &= \frac{(92.1^2 + 92.1^2 + 81.6^2 + \dots + 94.5^2)}{3} - \text{CF} \\ &= 1524.813 \end{aligned}$$

Step 10. Calculate Error(b) sum of squares.

$$\begin{aligned} \text{Error(b) SS} &= \text{Subplot SS} - \text{A X B SS} - \text{B SS} - \text{Error(a) SS} - \text{A SS} - \text{Rep SS} \\ &= 1524.813 - 40.687 - 706.88 - 111.749 - 443.689 - 143.465 = 78.343 \end{aligned}$$

Step 11. Calculate C sum of squares.

$$\begin{aligned} C\ SS &= \frac{\sum Y_{..l}^2}{rab} - CF \\ &= \frac{(634.1^2 + 744.3^2 + 849.0^2)}{4 \times 3 \times 2} - CF \\ &= 962.335 \end{aligned}$$

Step 12. Calculate A X C sum of squares.

$$\begin{aligned} A \times C\ SS &= \frac{\sum Y_{.jl}^2}{rb} - CF - A\ SS - C\ SS \\ &= \frac{(218.3^2 + 234.0^2 + 181.3^2 + \dots + 257.2^2)}{4 \times 2} - CF - A\ SS - C\ SS \\ &= 13.1097 \end{aligned}$$

Step 13. Calculate B X C sum of squares.

$$\begin{aligned} B \times C\ SS &= \frac{\sum Y_{.kl}^2}{ra} - CF - B\ SS - C\ SS \\ &= \frac{(291.4^2 + 345.2^2 + 364.3^2 + \dots + 484.7^2)}{4 \times 3} - CF - B\ SS - C\ SS \\ &= 127.831 \end{aligned}$$

Step 14. Calculate AxBxC sum of squares.

$$\begin{aligned}
 \text{AxBxC SS} &= \frac{\sum Y_{jkl}^2}{r} - \text{CF} - \text{A SS} - \text{B SS} - \text{C SS} - \text{AxB SS} - \text{AxC SS} - \text{BxC SS} - \\
 &= \frac{(96.9^2 + 116.4^2 + 124.6^2 + \dots + 149.4^2)}{4} - \text{CF} - \text{A SS} - \text{B SS} - \text{C SS} - \text{AxB SS} - \text{AxC SS} - \text{BxC SS} \\
 &= 44.019
 \end{aligned}$$

Step 15. Calculate Error(c) sum of squares.

$$\begin{aligned}
 \text{Error(c) SS} &= \text{Total SS} - \text{AxBxC SS} - \text{BxC SS} - \text{AxC SS} - \text{C SS} - \text{Error(b) SS} - \text{Ax B SS} - \text{B SS} - \\
 &\quad \text{Error(a) SS} - \text{A SS} - \text{Rep SS} \\
 &= 168.498
 \end{aligned}$$

Step 16 ANOVA

SOV	df	SS	MS	F (A,B, C fixed)
Replicate	3	143.45	47.819	
A	2	443.689	221.844	A MS/Error(a) MS = 11.91**
Error(a)	6	111.749	18.626	
B	1	706.88	706.88	B MS/Error(b) MS = 81.21**
A X B	2	40.688	20.344	AxB MS/Error(b) MS = 2.34
Error(b)	9	78.343	8.705	
C	2	962.335	481.168	C MS/Error(c) MS = 102.80**
A X C	4	13.110	3.277	AxC MS/Error(c) MS = 0.70
B X C	2	127.831	63.915	BxC MS/Error(c) MS = 13.66**
A X B X C	4	44.019	11.005	AxBxC MS/Error(c) MS = 2.35
Error(c)	36	168.498	4.681	
Total	71	2840.606		

LSD's for Split-split Plot Arrangement

1. To compare two whole plot means averaged over all sub and sub-sub-plot treatments (e.g. a_0 vs. a_1).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(a) \text{df}} \sqrt{\frac{2\text{Error}(a)\text{MS}}{rbc}}$$

$$= 2.447 \sqrt{\frac{2(18.626)}{4 \times 2 \times 3}}$$

$$= 3.05$$

2. To compare two subplot means averaged over all whole and sub-sub plot treatments (e.g. b_0 vs. b_1).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(b) \text{df}} \sqrt{\frac{2\text{Error}(b)\text{MS}}{rac}}$$

$$= 2.262 \sqrt{\frac{2(8.705)}{4 \times 3 \times 3}}$$

$$= 1.57$$

3. To compare two sub-subplot means averaged over all whole and subplot treatments (e.g. c_0 vs c_1).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(c) \text{df}} \sqrt{\frac{2\text{Error}(c)\text{MS}}{rab}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4 \times 3 \times 3}}$$

$$= 1.27$$

4. To compare two subplot means (averaged over all sub-subplot treatments) at the same levels of the whole plot (e.g. a_0b_0 vs. a_0b_1).

$$LSD = t_{\frac{\alpha}{2}, \text{Err}(b)\text{df}} \sqrt{\frac{2\text{Error}(b)\text{MS}}{rc}}$$

$$= 2.262 \sqrt{\frac{2(8.705)}{4 \times 3}}$$

$$= 2.72$$

5. To compare two whole plot means (averaged over all sub-subplot treatments) at the same or different levels of the subplot (e.g. a_0b_0 vs a_1b_0 or a_0b_0 vs a_2b_1).

$$LSD = t_{AB} \sqrt{\frac{2[(b-1)\text{Error}(b)\text{MS} + \text{Error}(a)\text{MS}]}{rbc}}$$

and

$$t_{AB} = \frac{(b-1)\text{Error}(b)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(b)\text{df}} \right) + \text{Error}(a)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(a)\text{df}} \right)}{(b-1)\text{Error}(b)\text{MS} + \text{Error}(a)\text{MS}}$$

∴

$$t_{AB} = \frac{(2-1)(8.705)(2.262) + 18.626(2.447)}{(2-1)8.705 + 18.626}$$

$$= 2.388$$

and

$$LSD = 2.388 \sqrt{\frac{2[(2-1)8.705 + 18.626]}{4 \times 2 \times 3}}$$

6. To compare two sub-subplot means (averaged over all subplot treatments) at the same levels of the whole plot (e.g. a_0c_0 vs. a_0c_1).

$$\text{LSD} = t_{\frac{\alpha}{2}, \text{Err}(c) \text{df}} \sqrt{\frac{2\text{Error}(c)\text{MS}}{rb}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4 \times 2}}$$

$$= 2.20$$

7. To compare two whole plot means (averaged over all subplot treatments) at the same or different levels of the sub-subplot (e.g. a_0c_0 vs. a_1c_0 or a_0c_0 vs. a_2c_1).

$$\text{LSD} = t_{AC} \sqrt{\frac{2[(c-1)\text{Error}(c)\text{MS} + \text{Error}(a)\text{MS}]}{rbc}}$$

and

$$t_{AC} = \frac{(c-1)\text{Error}(c)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(c) \text{df}} \right) + \text{Error}(a)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(a) \text{df}} \right)}{(c-1)\text{Error}(c)\text{MS} + \text{Error}(a)\text{MS}}$$

\therefore

$$t_{AC} = \frac{(3-1)(4.681)(2.030) + 18.626(2.447)}{(3-1)4.681 + 18.626}$$

$$= 2.307$$

and

$$\text{LSD} = 2.307 \sqrt{\frac{2[(3-1)4.681 + 18.626]}{4 \times 2 \times 3}}$$

$$= 3.52$$

8. To compare two sub-subplot means (averaged over all whole plot treatments) at the same levels of the subplot (e.g. b_0c_0 vs. b_0c_1).

$$LSD = t_{\frac{\alpha}{2}, \text{Err}(c)df} \sqrt{\frac{2\text{Error}(c)MS}{ra}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4 \times 3}}$$

$$= 1.79$$

9. To compare two subplot means (averaged over all whole plot treatments) at the same or different levels of the sub-subplot (e.g. b_0c_0 vs. b_1c_0 or b_0c_0 vs. b_2c_1).

$$LSD = t_{BC} \sqrt{\frac{2[(c-1)\text{Error}(c)MS + \text{Error}(b)MS]}{rac}}$$

and

$$t_{BC} = \frac{(c-1)\text{Error}(c)MS \left(t_{\frac{\alpha}{2}, \text{Err}(c)df} \right) + \text{Error}(b)MS \left(t_{\frac{\alpha}{2}, \text{Err}(b)df} \right)}{(c-1)\text{Error}(c)MS + \text{Error}(b)MS}$$

∴

$$t_{BC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142 \sqrt{\frac{2[(3-1)4.681 + 8.705]}{4 \times 3 \times 3}}$$

$$= 2.15$$

10. To compare two sub-subplot means at the same combination of whole plot and subplot treatments (e.g. $a_0b_0c_0$ vs. $a_0b_0c_2$).

$$LSD = t_{\frac{\alpha}{2}, \text{Err}(c)\text{df}} \sqrt{\frac{2\text{Error}(c)\text{MS}}{r}}$$

$$= 2.030 \sqrt{\frac{2(4.681)}{4}}$$

$$= 3.11$$

11. To compare two subplot means at the same level of whole plot and sub-subplot (e.g. $a_0b_0c_0$ vs. $a_0b_1c_0$).

$$LSD = t_{ABC} \sqrt{\frac{2[(c-1)\text{Error}(c)\text{MS} + \text{Error}(b)\text{MS}]}{rc}}$$

and

$$t_{ABC} = \frac{(c-1)\text{Error}(c)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(c)\text{df}} \right) + \text{Error}(b)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(b)\text{df}} \right)}{(c-1)\text{Error}(c)\text{MS} + \text{Error}(b)\text{MS}}$$

∴

$$t_{ABC} = \frac{(3-1)(4.681)(2.030) + 8.705(2.262)}{(3-1)4.681 + 8.705}$$

$$= 2.142$$

and

$$LSD = 2.142 \sqrt{\frac{2[(3-1)4.681 + 8.705]}{4 \times 3}}$$

$$= 3.72$$

- 12 To compare two whole plot means at the same combination of subplot and sub-subplot treatments ($a_0b_0c_0$ vs. $a_1b_0c_0$).

$$\text{LSD} = t_{ABC} \sqrt{\frac{2[(b)(c-1)\text{Error}(c)\text{MS} + (b-1)\text{Error}(b)\text{MS} + \text{Error}(a)\text{MS}]}{rbc}}$$

and

$$t_{ABC} = \frac{(b)(c-1)\text{Error}(c)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(c)\text{df}} \right) + (b-1)\text{Error}(b)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(b)\text{df}} \right) + \text{Error}(a)\text{MS} \left(t_{\frac{\alpha}{2}, \text{Err}(a)\text{df}} \right)}{(b)(c-1)\text{Error}(c)\text{MS} + (b-1)\text{Error}(b)\text{MS} + \text{Error}(a)\text{MS}}$$

∴

$$t_{ABC} = \frac{((2 * (3 - 1)(4.681)(2.030) + (2 - 1)8.705(2.262) + 18.626(2.447))}{(2 * (3 - 1)4.681) + (2 - 1)8.705 + 18.626}$$

$$= 2.242$$

and

$$\text{LSD} = 2.242 \sqrt{\frac{2[(2 * (3 - 1)4.681) + (2 - 1)8.705 + 18.626]}{4 \times 2 \times 3}}$$

$$= 4.39$$