Investigating the interplay of students' mathematics and physics thinking in Linear Algebra and Quantum Mechanics

Caitlin E. Rainey, Warren M. Christensen
Middle Tennessee State University, North Dakota State University

Theoretical Framework

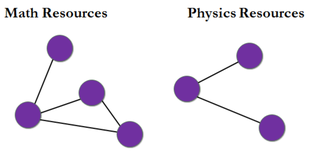
Through this research, we hope to identify aspects of student math and physics framing. A resources framework models knowledge as existing in pieces and these pieces are compiled in real-time to form an explanation. The notion of framing describes the idea that these resources are context dependent and a person will bring to bear those resources that they think are useful for answering the question.

Research Questions

How do students and faculty:

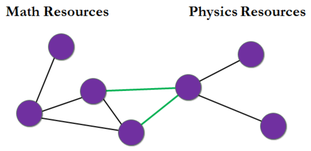
- use math language when solving math and physics problems?
- reason while solving math and physics problems?
- frame math and physics problems and can we identify them?

Framework



Toy model of student conceptual resources that are not connected and either of which might be called on when starting a problem.

Experts are often considered to have more connections between math and physics resources and can move between frames more readily.



Nontraditional PER Interview Method

Our data were collected through video recorded interviews. These interviews lasted an average of 60 minutes where the interviewee wrote their responses to the questions in the protocol on a provided whiteboard.

The interviewees consisted of:

- Two Physics Professors
- Two Physics Graduate Students
- Three Physics Undergraduate Students

Unlike most Physics Education Research (PER), we are interviewing physics professors along with the students (graduates and undergraduates). This method, however, is more prevalent in Research in Undergraduate Mathematics Education (RUME).

Interview Protocol

This is a three part protocol written in a nontrivial, progressive order. Each part of the protocol consists of prompts with follow-up questions.

Part I: Math Problem written by a mathematician
Part II: Math problem written by a physicist
Part III: Physics problem written by a physicist

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Part I Prompt

$$\begin{bmatrix}1 & 3\\ 6 & -1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$$

⬇

Consider a 2x2 matrix A and a vector $\begin{bmatrix}x\\y\end{bmatrix}$.

How do you think about $A\begin{bmatrix}x\\y\end{bmatrix} = 2\begin{bmatrix}x\\y\end{bmatrix}$?

⬇

Suppose $A=\begin{bmatrix}4 & 2\\1 & 3\end{bmatrix}$.
Now how do you think about $A\begin{bmatrix}x\\y\end{bmatrix} = 2\begin{bmatrix}x\\y\end{bmatrix}$?

Part I: Math Problem written by a Mathematician:

Q: "How do you think about what the equals sign means when you see it written in the context of this equation?"

Part II: Math Problem written by a Physicist:

Q: "Given the following 2x2 matrix, Is it possible to determine the eigenvalue(s) for this matrix?

Part II Prompt

$$\mathbf{M} = \begin{bmatrix}2 & 1\\0 & 3\end{bmatrix}$$

Part III Prompt

An electron is in the spin state $\chi = \frac{1}{5}\begin{bmatrix}4\\3\end{bmatrix}$.

⬇

$$S_x = \frac{h}{2}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

Part III: Physics Problem written by a Physicist:

Q: "If we wanted to determine the probability that the particle was either spin up or spin down in the z-direction, can you describe how would we go about doing that?"

References

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- Henderson, F., Rasmussen, C., Sweeney, G., Wawro, M., & Zandieh, M. (2010). Symbol sense in linear algebra: A start towards eigentheory. InProceedings of the 13th Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education Conference on Research in Undergraduate Mathematics Education..(Accessed:).

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Common Question (CQ)

Each part of the protocol has a common question intended to identify features that influence students and faculty framing.

"After doing this problem, do you feel you're doing more math or doing more physics? Why?"

Responses to the Common Question

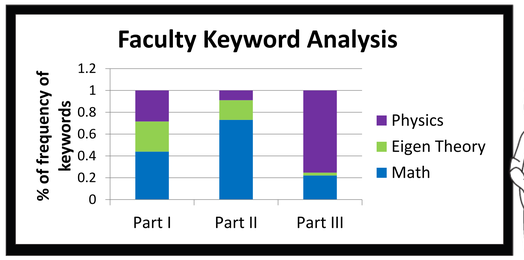
Below is the analysis of the math and physics language used throughout the interview. The frequency of the math and physics language is compared to the responses to the common question to gain further insight into how they frame the problem.

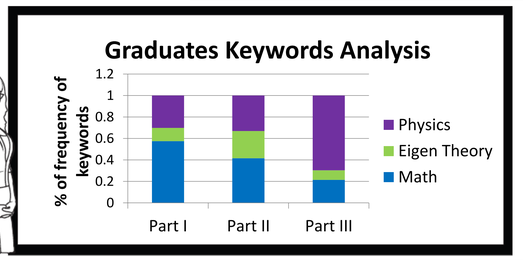
Examples of Keywords

Math Keywords	Eigen Theory Keywords	Physics Keywords
Matrix	Eigenvector	Quantum Mechanics
Vector	Eigenvalue	Hermitian
Matrix Dimension (2x2)	Characteristic equation	Spin/ Spin State
Equal		Bra-Ket (notation)
System of Equations		ħ/2, ħ
Orthogonal		Wavefunction

Response to CQ:

- Part I: Math
- Part II: Math
- Part III: Neither



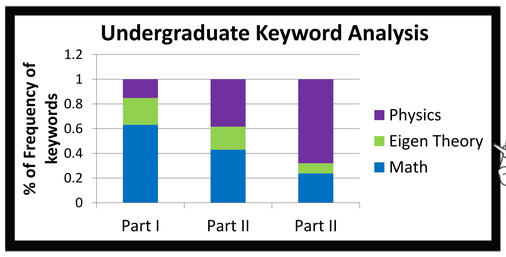


Response to CQ:

- Part I: Math
- Part II: Physics
- Part III: Physics

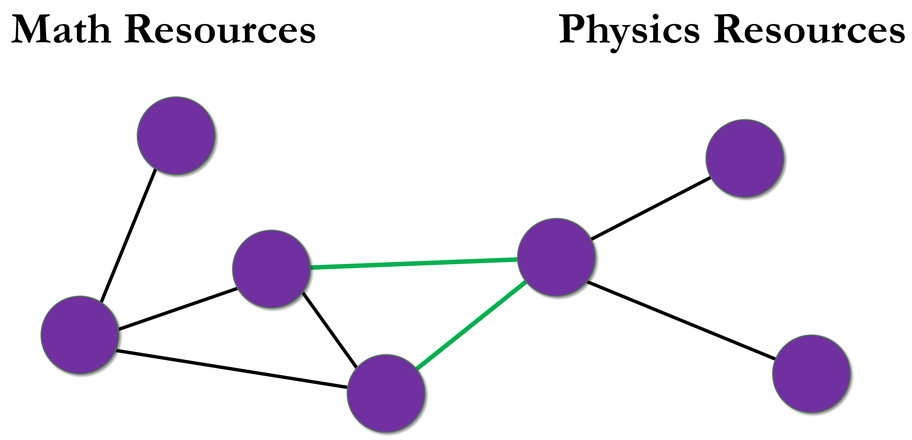
Response to CQ:

- Part I: Math
- Part II: Math
- Part III: Math



Conclusion

From the perspective of framing, we observe a disconnect between what our interview subjects report "doing" and the language and reasoning they use within the interview settings. Both experts and novices struggle to identify the types of thinking in which they are engaging.


Math Resources
Physics Resources