

# Intermittency Analysis in Chaotic Systems

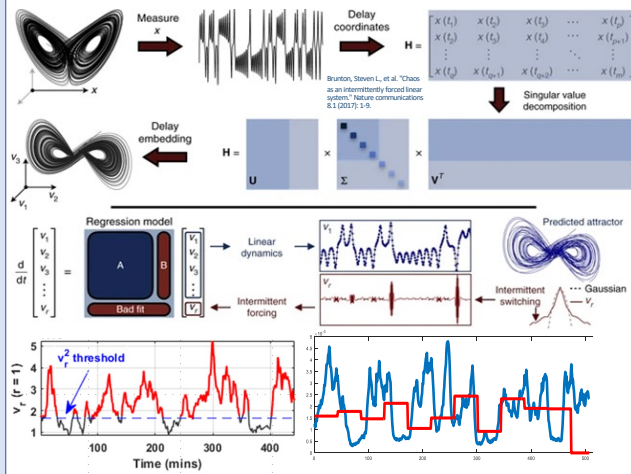
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## Introduction

Complex physiological processes have commonly exhibited chaotic dynamics and intermittent phenomena. Such complex systems have been analyzed and decomposed into intermittently forced linear systems by means of the Hankel alternative view of Koopman (HAVOK) method. Furthermore, wavelet analysis methods have been used to further analyze the spectral and temporal properties of the chaotic intermittent bursting in systems such as apneic events in obstructive sleep apnea (OSA) patients. In this paper, we present further methods to analyze the intermittent bursting in these systems, using OSA as a case study to validate our results. Results show that our methods improve the correlation for all patients, implying that filling these gaps in the previous research will help better control this advanced pathophysiological process for better preventative treatment.

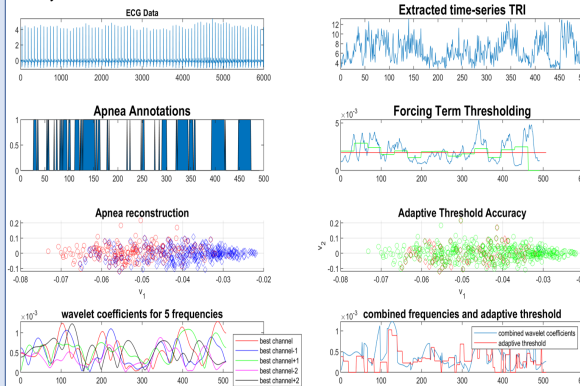
## Methods

The first step is to reconstruct the state space using and create a forced linear dynamical system representation using HAVOK. Then, in the intermittency analysis, we perform the continuous wavelet transform, identify the best fitting frequency, and combine its information with neighboring frequencies. Next, we perform the adaptive thresholding method on both the forcing component and the wavelet coefficients by computing the maximum correlation for each window size, and selecting the optimum window length. Finally we combine the best wavelet frequency with information from its neighboring frequencies to form a broad frequency band.

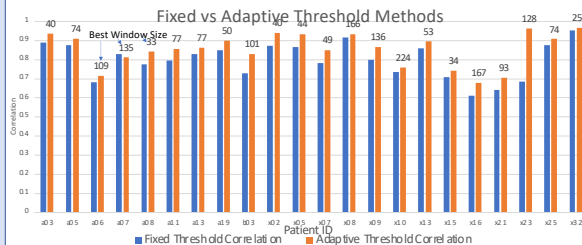


## Results

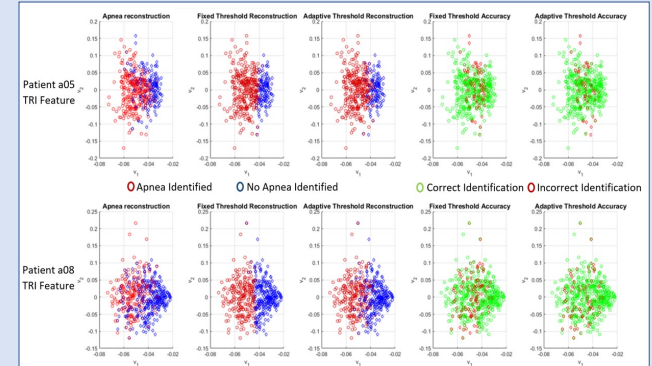
This figure shows the methods applied to patient a08 using TRI as the extracted feature. It shows the raw ECG signal, extracted time-series  $x(t)$ , the OSA annotations, the intermittent forcing term with both fixed and adaptive threshold, the diffeomorphic attractor created by the apnea annotations, the reconstruction accuracy of the adaptive threshold method, the optimum and surrounding wavelet frequency bands, and the combined wavelet analysis.



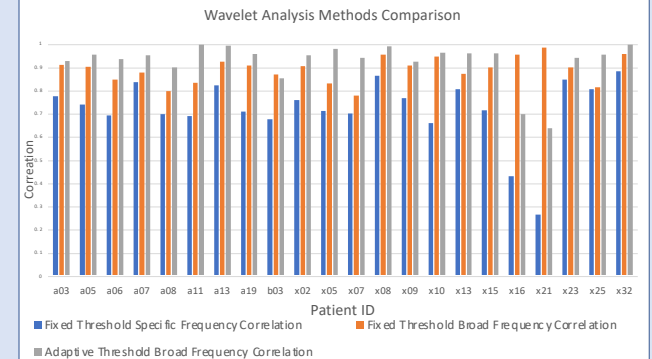
We performed the fixed threshold and adaptive threshold methods on 22 patients, using the best reconstruction feature for each. We then compared the maximum correlations of each method, as well as the best window size to maximize the correlation of the adaptive threshold method. The results are summarized in this chart



When analyzing the attractors reconstructed using these two methods, it is easy to see the limitations of the fixed threshold method as compared to the adaptive threshold method. The figure in the top right shows how the adaptive threshold method is better able to capture information from the mixing of the two modes of the system.



Our next task was to improve the accuracy of the wavelet coefficient thresholding methods. To do this we combined information from the best performing frequency with its neighboring frequencies, forming a broad frequency band. We first used this method using the fixed threshold, and then combined it with the adaptive threshold method. The results are summarized in this chart.



## Conclusion

We analyzed a new method for thresholding the  $v_1$  forcing term in the OSA case study and found that the adaptive threshold consistently performed better than the fixed threshold, showing that this new method can be used to more accurately predict apnea events based off the amplitude of the intermittent forcing. The broad frequency band analysis also improved upon the correlations of the wavelet analysis methods. Our methods should motivate further study towards controlling and predicting other pathophysiological processes that exhibit these characteristics, and eventually other important systems in science and engineering that have yet to be fully explored.