Slope Failures and Calculus

Content Area(s)/Course/Grade: AP Calculus	Unit: Techniques of Integration
Lesson Topic: Area Under a Curve	Length of Lesson: 2-3 Days
Materials for Students: Calculator and writing utensil	Materials for Teacher: Grid plates and foam cutouts (see pictures), guided worksheet/notes

Standard(s) Addressed:
Approximating Areas with Riemann Sums
Interpreting the Behavior of Accumulation Functions Involving Area

Student Outcome(s):
I can approximate area under a curve using Riemann Sums
I can find the area under a curve using geometric formulas
I can interpret the behavior and meaning of functions involving area (examples: total rainfall or total land)

Context for Learning

This is the start of a unit on integration and how we first learn to approximate the value before we can find it exactly. The addition of Slope Failures and Rain Fall data give the lesson more meaning and help students become more engaged and build a stronger understanding and appreciation for the material.

Instructional Delivery

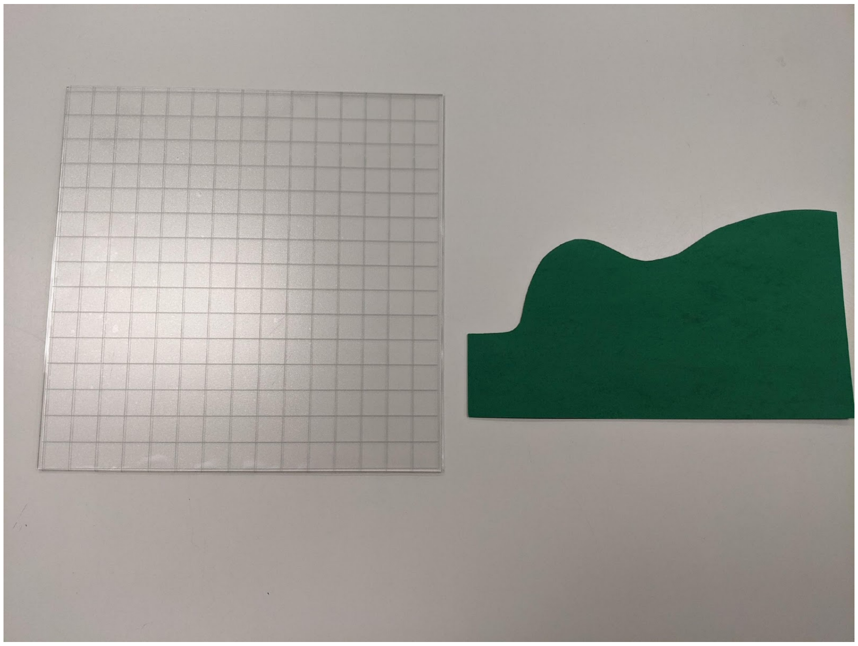
Lesson notes: This lesson will begin with an explanation of slope failures. From there students will break into groups to work on an activity for the next 15 minutes where they find area under the curve. When they all finish we will discuss their findings as a class for about 5 minutes. The last 20 minutes will be time for notes. The 2nd day will begin with a discussion about what they thought could cause these slope failures. Hopefully a student will talk about rain fall which will guide the next activity they do. They will do a second activity that day using what they learned from the notes from the day before for about 20 minutes. The last 20 minutes will be work time on worksheet. A third day may be needed if activities or discussion went long or more work time on the worksheet is needed with teacher guidance. (notes are attached)

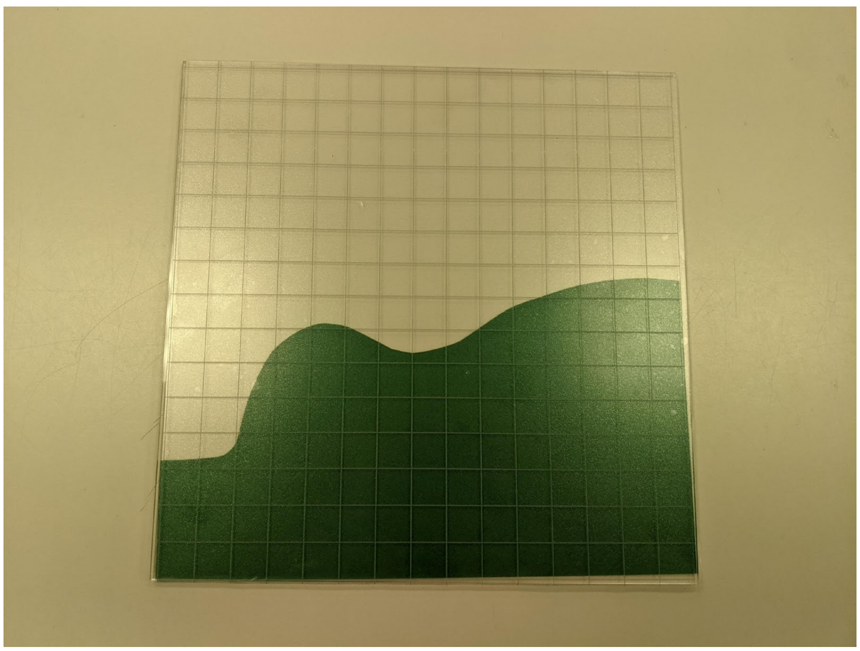
Activity: Students will be calculating area under a curve made out of foam with coordinate grid on plexiglass or plastic (need to be see through). During the second activity which is similar to the first they will use the concepts they developed and learn in the notes to complete it instead. (activity worksheet is attached)

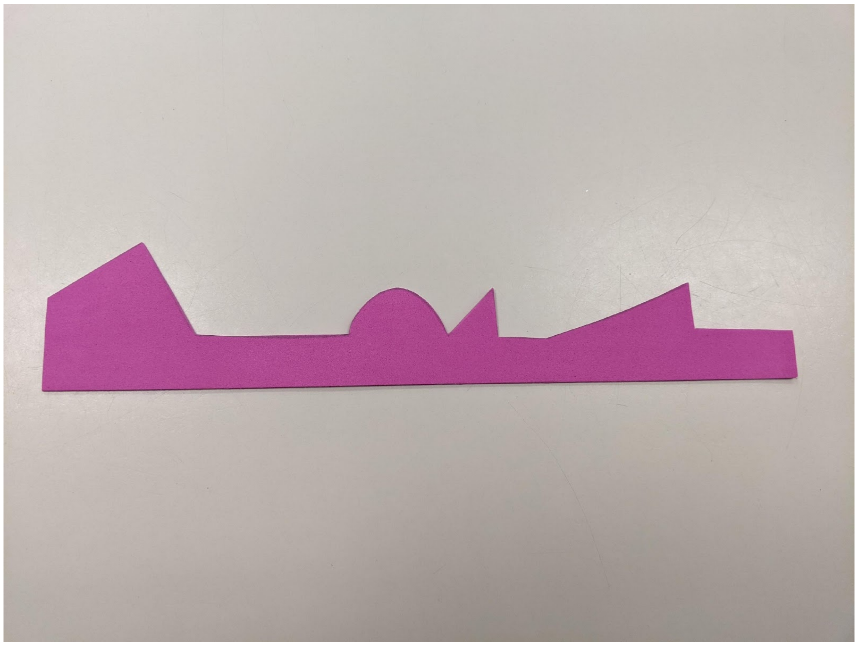
Assessment/Evaluation (Formative/Summative)

There will be an informal formative assessment in the form of their worksheet. Gather how they are doing by walking around to each student and observing them work. There will be a formal formative assessment the day after this lesson in the form of a mini quiz that will be one Reimann Sums question solve all four ways. (worksheet and mini quiz will be attached with this lesson plan)

Accommodations: Make sure everyone has a calculator for this worksheet and activity. Reduce the amount of question for those who need it. Walk around and help those students that need more help. Extra time for students that need it on the mini quiz.







Slope Failures and Calculus
Lesson 1 Area Under a Curve

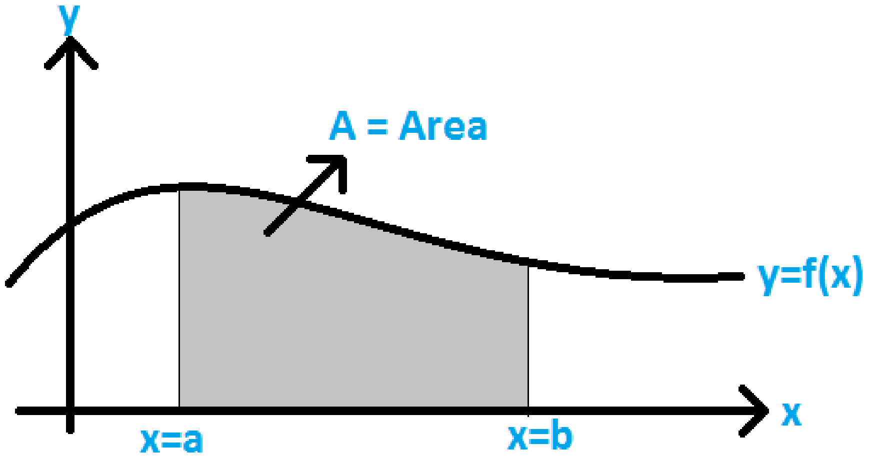
Objectives: To be able to use Riemann Sums and geometric formulas to find area under a curve. To be able to interpret the behavior and meaning of functions involving area. To be able to understand basic idea of Slope Failures and possible causes.

Pre-Assessment on Slope Failure

What is a Slope Failure?

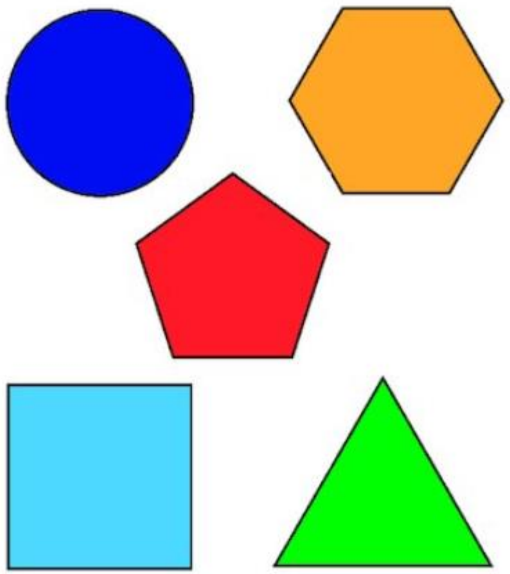
Area Under the Curve Activity

- You will need to break into groups of 3-4 people.
- You will need to grab one activity packet per group
- You will need to grab one plastic measuring sheet
- You will need to grab one of each foam cut out.
- You may use a calculator on this activity



What did your group do / What did you learn?

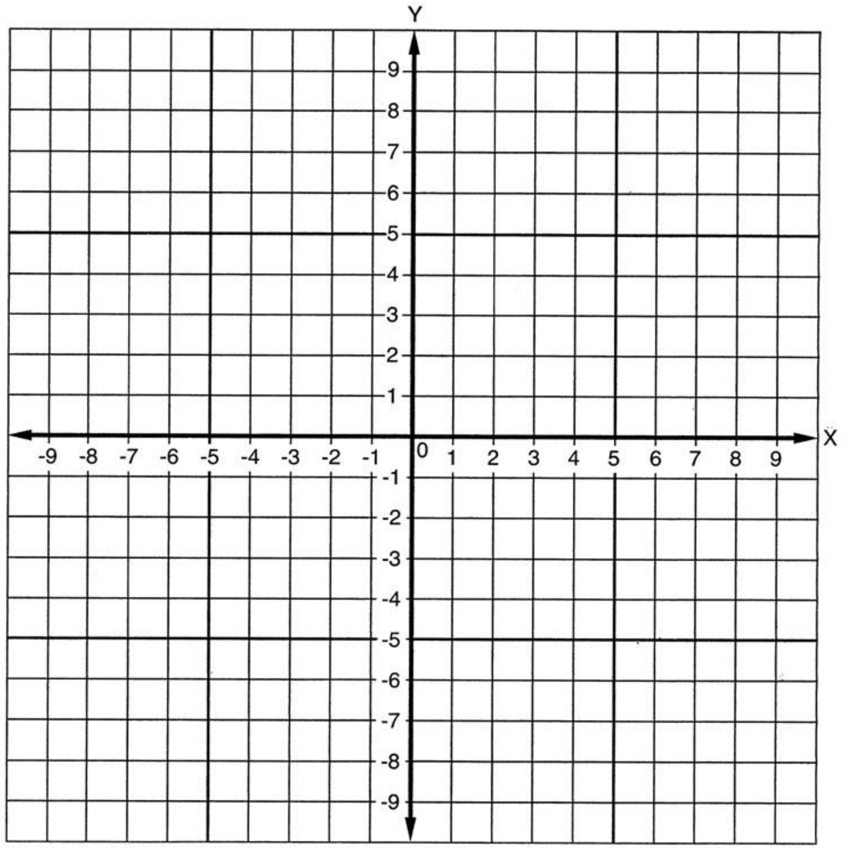
- What techniques did you use?
- Did you stay with the same technique for all the questions?
- Was your technique accurate?
- Would you do something differently now?
- How could we improve the techniques?



The application of derivatives on a graph were that they were the slope of a tangent line to a curve. Graphically, the application of antiderivatives is the area under a curve bounced by the x-axis.

Find:

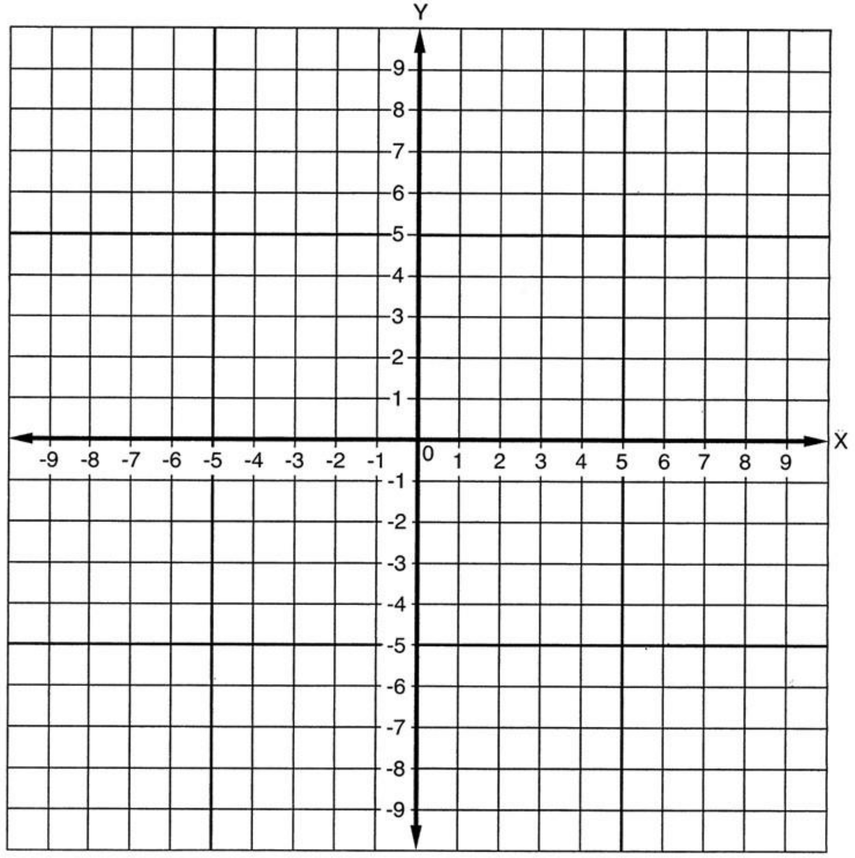
$$\int_0^4 2x dx$$



Some areas you can easily find exactly using are formulas like triangles, rectangles, or circles, but often, the are under a curve is not one of these regions.

$$\int_{-1}^{2} x^3 + 2$$

There are 4 commonly used estimation techniques to approximate this area.



We are going try using all four techniques on the same function.

Estimate using 4 divisions

$\int_0^2 -x^2 + 7dx$

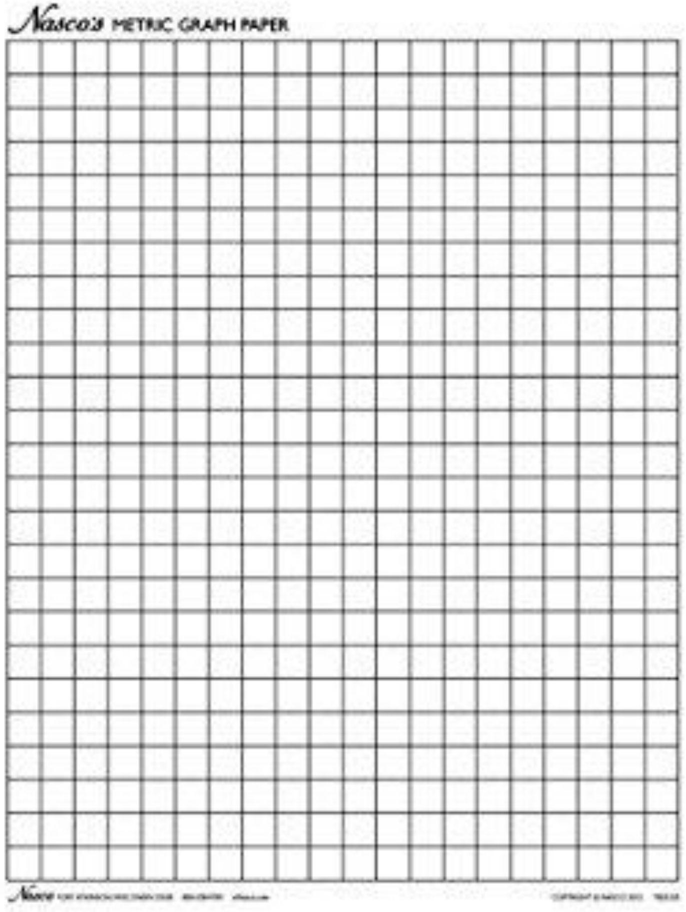
First method is Right Sum Rule:

We divide the region into a set number of

Divisions and then find the value at the right of

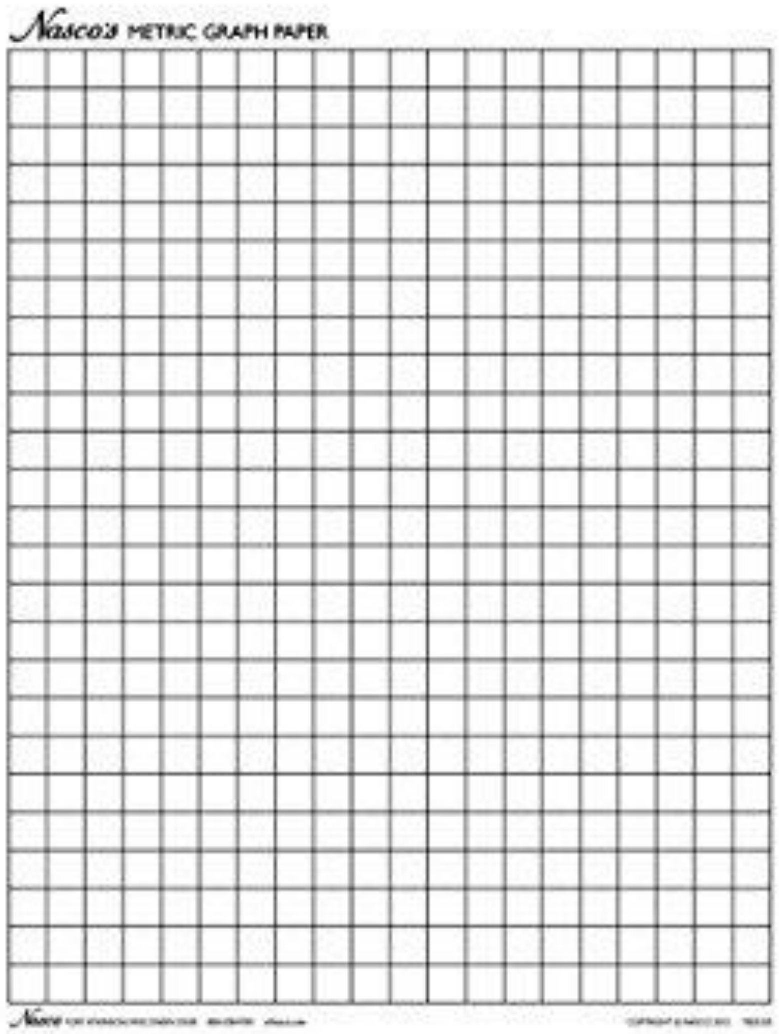
Each division and find the area of each of those

Divisions to estimate the actual area.



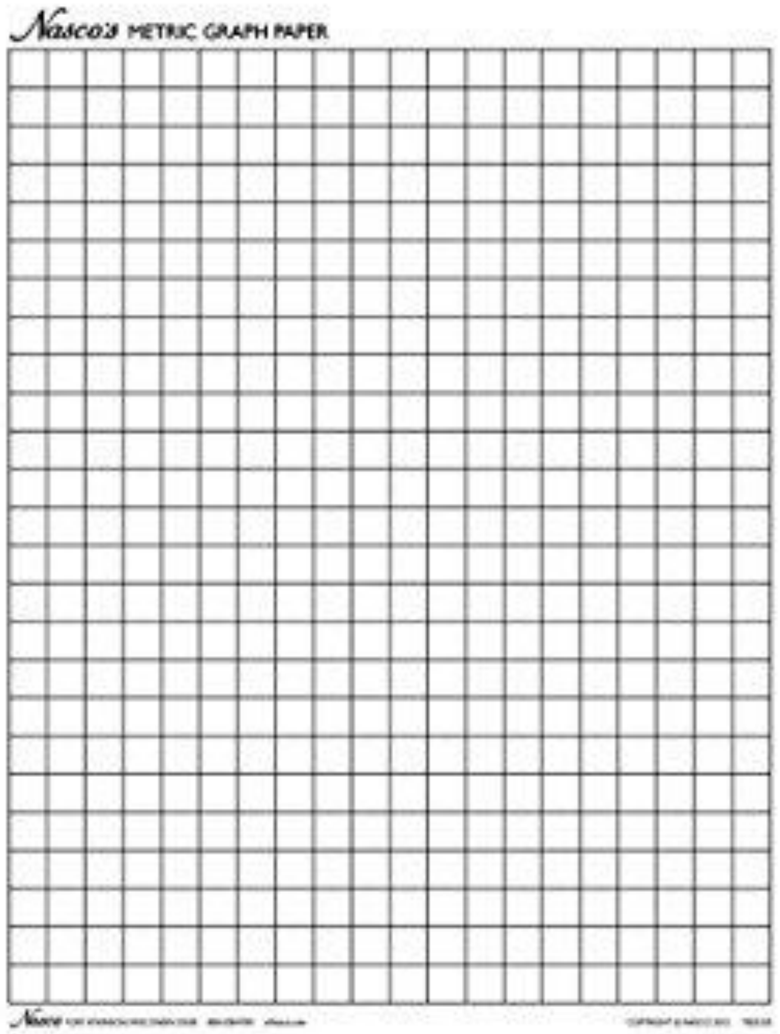
Try the same function with the Left Sum Rule:

$\int_0^2 -x^2 + 7dx$



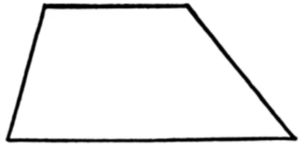
Try the same function with the Midpoint Rule:

$$\int_0^2 -x^2 + 7dx$$

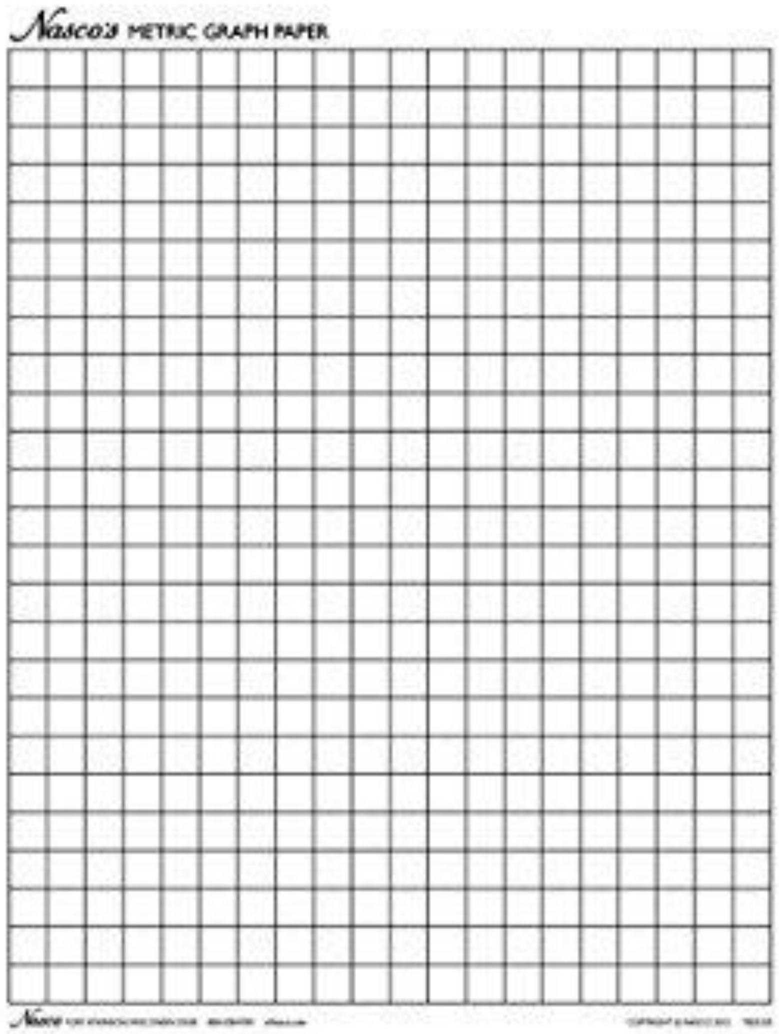


Try the same function with the Trapezoidal Rule:

$$\int_0^2 -x^2 + 7dx$$


Trapezoid.

Here we make each division into a trapezoid to approximate the area.



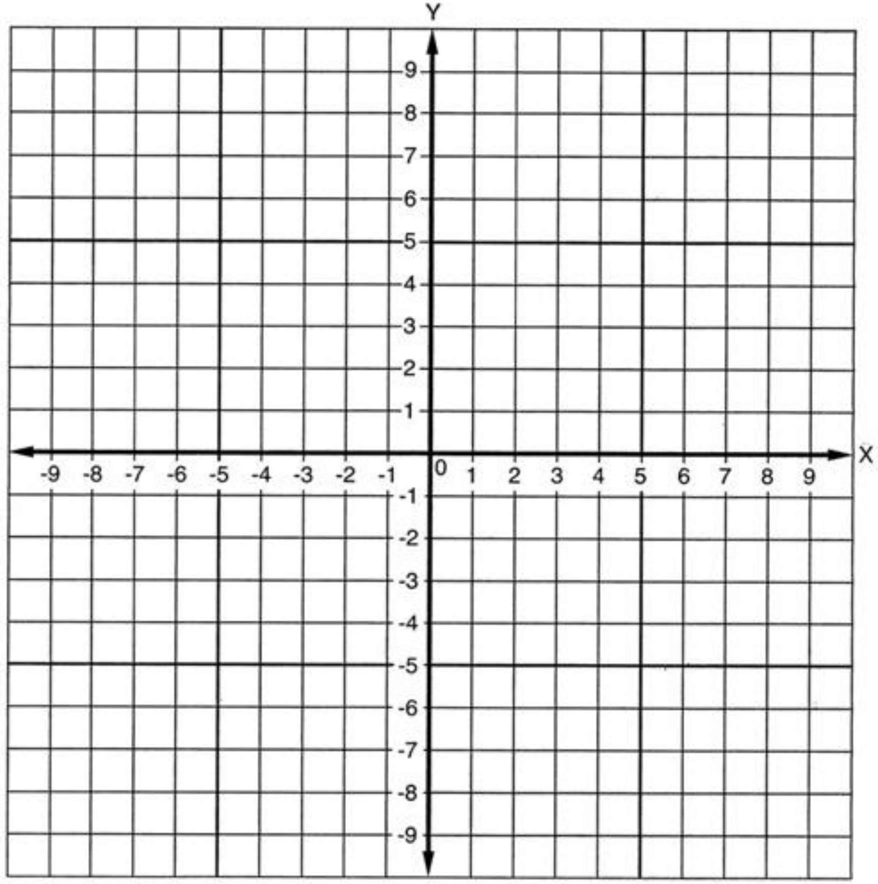
How can you improve your approximation of the area?

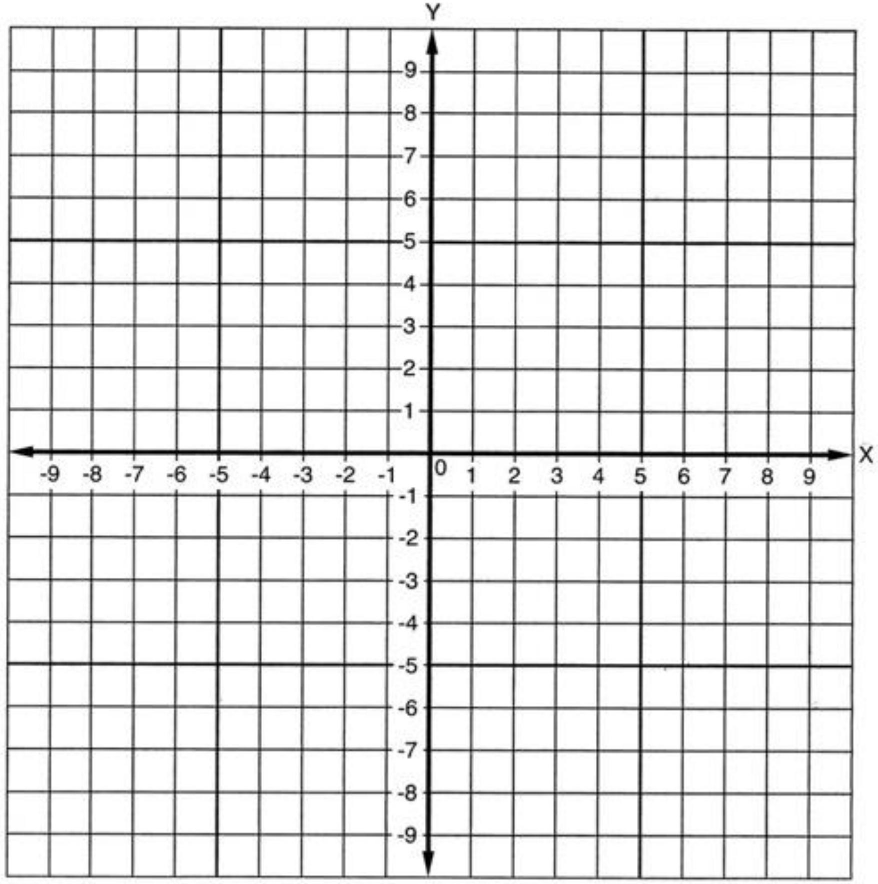
One Technique is to use formulas for exact geometric shapes.

Find these exact areas using geometric formulas:

$\int_{-3}^{4} 5dx$

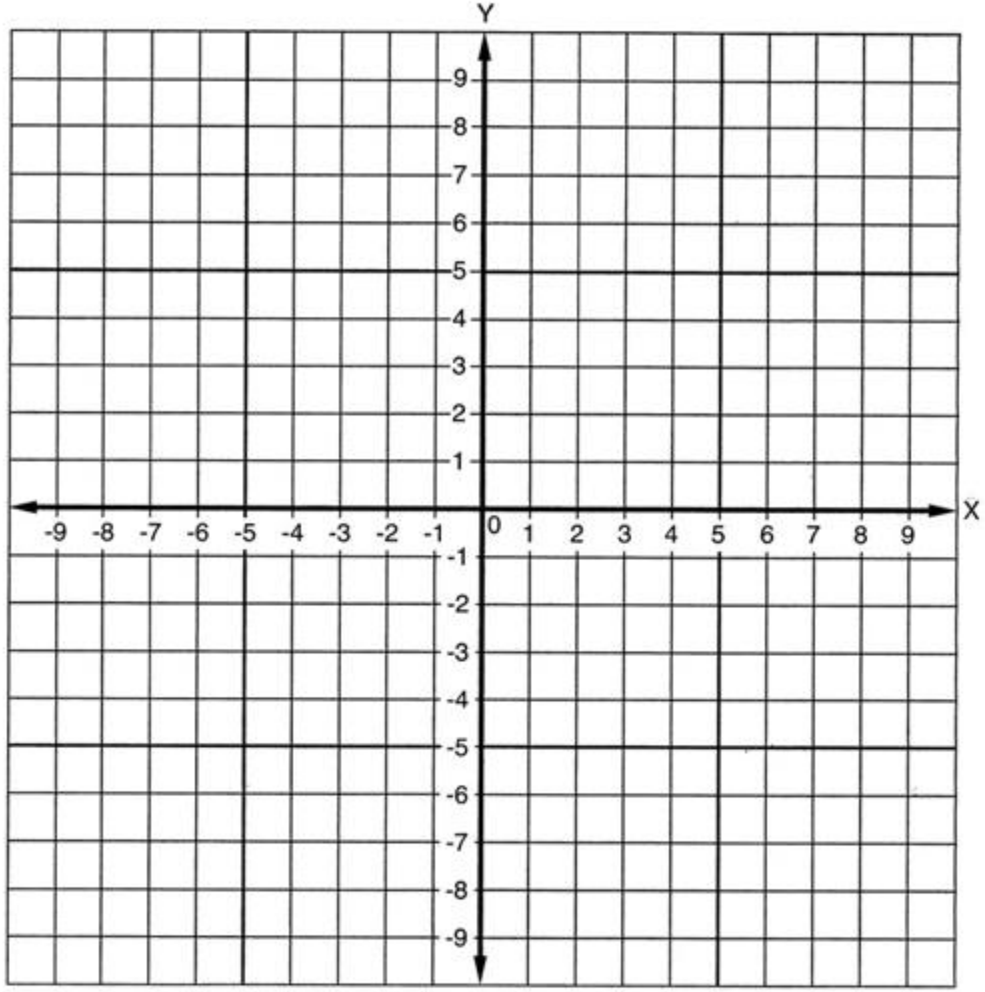
$\int_{1}^{4} (2x - 1)dx$



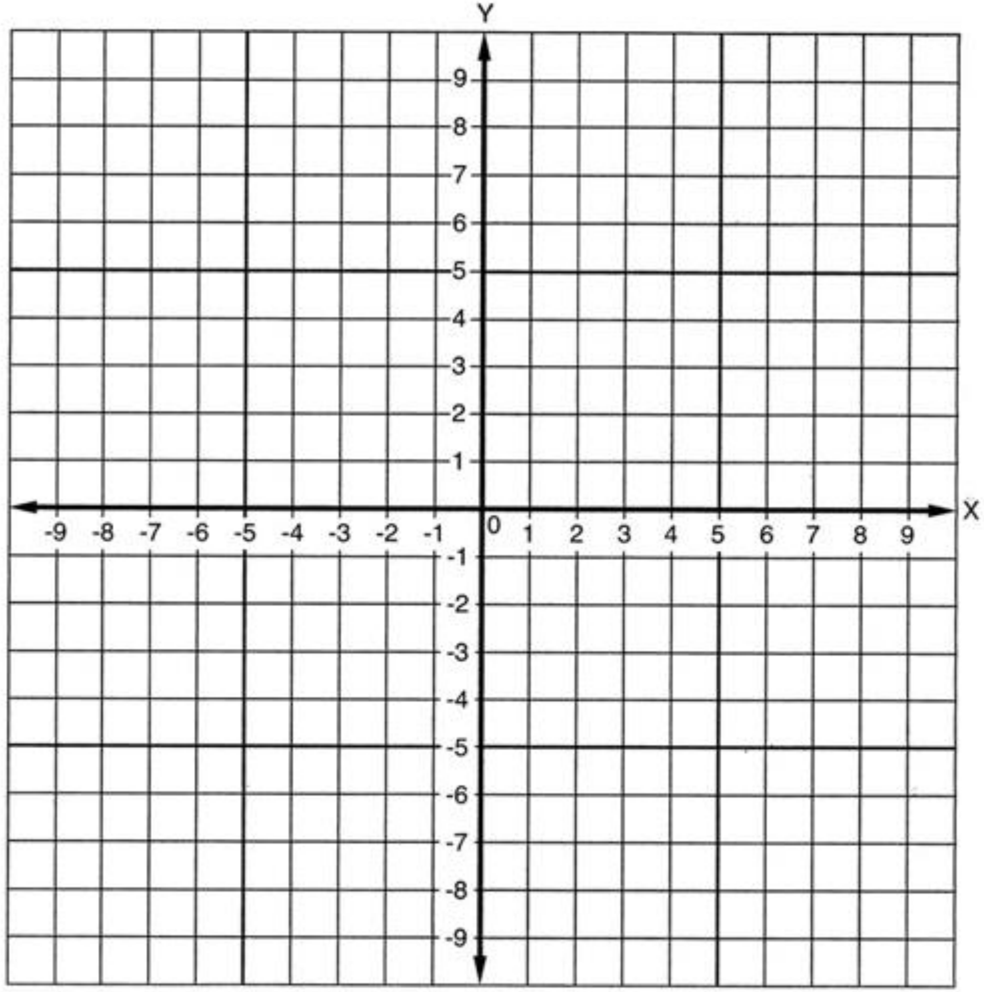


Find these exact areas using geometric formulas:

$\int_{-3}^{3} \sqrt{9 - x^2}$



$\int_{-3}^{3} (3 - |x|)dx$



Slope Failures and Calculus Lesson 1 Area under a curve

Hand on Activity Day 1 Names of the group:_______________________

Slope failures happen all across the world and often have terrible effects. We will learn more about slope failures as this unit progresses and how there is lots of calculus connected to the concept. In today activity your group you will need to work together, discuss possible techniques, and find the area of the foam sheets using the plastic sheet and the coordinate plane on it. There is no one correct way to find these so please use some creative and critical thinking.

Foam sheet 1 (color)

What is the area of this foam sheet?

How did you find that area explain well enough that I would be able to replicate your method and get the same results as you?

Foam sheet 2 (color)

What is the area of this foam sheet?

How did you find that area explain well enough that I would be able to replicate your method and get the same results as you?

Foam sheet 3 (color)

What is the area of this foam sheet?

How did you find that area explain well enough that I would be able to replicate your method and get the same results as you?

Slope Failures and Calculus Lesson 1 Area under a curve

Hand on Activity Day 2 Names of the group:______________________

Slope failures happen across the world and often have terrible effects. We will learn more about slope failures as this unit progresses and how there is lots of calculus connected to the concept. In today activity your group you will need to work together, discuss possible techniques, and find the area of the foam sheets using the plastic sheet and the coordinate plane on it. The foam sheets now represent rainfall data as we discussed that is one of the main causes of slope failures. Today we will use the two techniques we learned yesterday to find the area of the foam sheets.

Foam sheet 1 (color)

Use the Techniques of Reimann Sums and find the area using all four types with 8 divisions.

What do you think the actual total rainfall amount is?

Which of the Reimann Sums techniques do you think is the more accurate?

Foam sheet 2 (color)

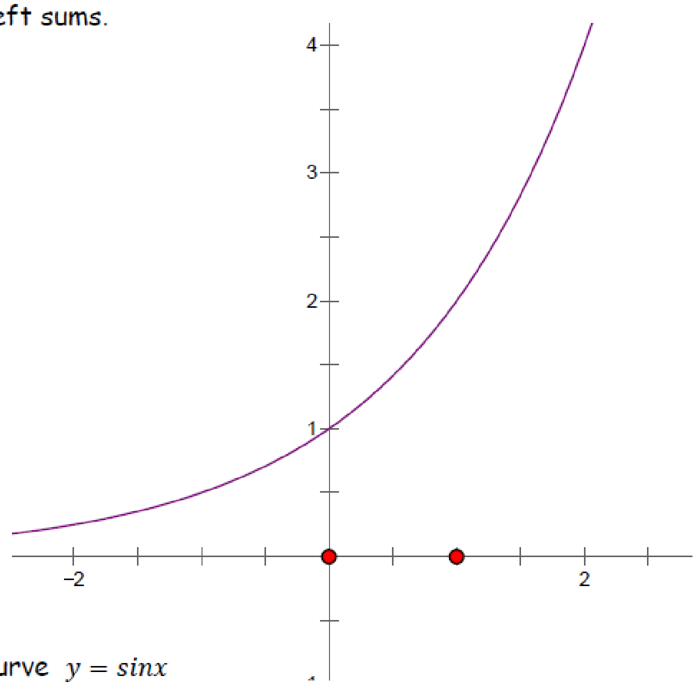
Find the area of the foam sheet and thus the total rainfall by using exact geometric formulas.

Slope Failure and Calculus   Lesson 1   Name:____________________

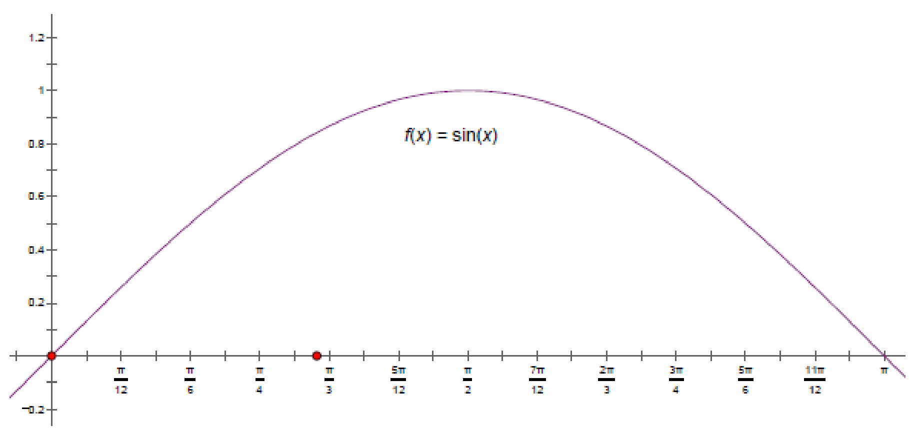
Complete the following questions.

Estimations of Areas Under Curves-Show your work!!! I should be able to see the sum of all your rectangles/trapezoids!! **Drawing in the rectangles is not necessary.**

1. Estimate the area under the curve using right and left sums for $f(x) = 2^x$ from [-2,2] using six equal divisions.
   Label your two answers as right or left sums.



2. Estimate the area under the given curve $y = sinx$
   Using the midpoint and trapezoid rule with
   Four equal divisions from [0,π].



3. Find given area under the function $f(x) = \frac{1}{x}$ bounded by the x-axis from $x = 1$ to $x = 2$, using 5 subintervals. Show your work!

a. Right Sum:

b. Left Sum:

c. Midpoint Rule:

d. Trapezoidal Rule:

4. Sketch the region represented by the definite integral and using GEOMETRY FORMULAS (not right sum, left sum, trapezoidal rules) evaluate the area of the region exactly.

a. $\int_0^3 4dx$

b. $\int_{-a}^{a} 4dx, a > 0$

c. $\int_0^2 (2x+5)dx$

d. $\int_{-1}^{1} (1-|x|)dx$

e. $\int_0^8 (8-x)dx$

f. $\int_{-3}^{3} \sqrt{9-x^2}dx$

5. When is a right sum rule always an overestimate? An underestimate?

6. When is the left sum rule always an overestimate? An underestimate?

7. Which rule is the best, explain your reason?

8. What do you know about slope failures, what conditions do you think possibly affect them?

SF and Calc Lesson 1 Mini Quiz   Name: ________________

Apply Right, Left, and Trapezoid method of Reimann's Sums to approximate the area under f(x)= $2x^2 - x + 1$ using 4 subdivisions with equal width on the interval [1,5]

SF and Calc Lesson 1 Mini Quiz   Name: ________________

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